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On Stability in Competition: Tying and Horizontal Product Differentiation

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Abstract

We combine Hotelling's model of product differentiation with tie-in sales. A monopolist in one market competes with another firm in a second market. In equilibrium firms choose zero product differentiation. Due to the tying structure no firm can gain the whole market by a small price reduction. A differentiation effect due to tie-in sales leads to this equilibrium stability.

Keywords: Horizontal product differentiation, Hotelling, tie-in sales, equilibrium existence.
JEL-Classification Numbers: $D43, L10, L11, L13$.

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1 Introduction

We address equilibrium existence for Hotelling's model of horizontal product differentiation. To address equilibrium existence for Hotelling's model we combine it with tie-in sales. Tie-in sales require consumers to buy a good as a condition for buying another good.[1]

Examples for tie-in sales motivate the widely used setting of a monopolist in one market competing with another firm in a second market. In the second market firms offer homogenous or given differentiated products. We modify this basic framework by modeling the second market in Hotelling's way. Thus, our model endogenizes firms' differentiation choices.

The combination of horizontal product differentiation with tie-in sales results in zero differentiation. In equilibrium, the firms' competitively supplied goods are homogeneous. Yet no firm attracts the entire market by a small price reduction. The tying firm does not serve consumers with low valuations for the monopoly good. The non-tying firm cannot win the entire market with a price reduction such that its price is non-negative. Not all of the tying firm's consumers give up the monopoly good for a price reduction.

Our model and its outcome are closely related to the work by Carbajo, de Meza, and Seidmann (1990) as well as Martin (1999). As is common in the tying literature both Carbajo, de Meza, and Seidmann as well as Martin assume that the bundling firm is a monopolist in one market and faces competition from another firm in another market. In the duopoly market the firms' goods are homogeneous. While the two analyses agree about the market structure, they differ in the competition mode. In the model by

[1] A survey on tie-in sales goes beyond the scope of this paper. Therefore, we refer to the extensive literature on tie-in sales and bundling for detailed definitions and examples. References on definitions and examples are, e.g., Burstein (1960, 1988), Adams and Yellen (1976), and Whinston (1990).

Carbajo, de Meza, and Seidmann firms compete in prices. By contrast, Martin analyzes a model with quantity competition.

The main finding of the analysis by Carbajo, de Meza, and Seidmann is that imperfect competition creates a strategic incentive for bundling. Bundling alters the behavior of the monopolist's rival and reduces competitiveness in the duopoly market. Specifically, if the monopolist bundles, it no longer sells to all consumers. It is profitable to serve only consumers with high valuations for the monopoly good. This in turn causes the monopolist's rival to act less aggressively. Bundling itself creates product differentiation. Hence, the bundle and the competitively offered good alone are not homogeneous. Both firms can raise prices above costs.

Martin's main result is similar to that of Carbajo, de Meza, and Seidmann: Bundling has a strategic effect because it changes the substitution relation for goods bought by consumers. The result is for the case when goods are independent in demand. Even in this case, the decision to bundle links the two goods. The two independent goods become substitutes.

Our analysis is closer related to Carbajo, de Meza, and Seidmann than to Martin. Like Carbajo, de Meza, and Seidmann we consider competition in prices. In line with Carbajo, de Meza, and Seidmann, we find that bundling softens competition.[2] This competition-softening mechanism is responsible for equilibrium stability in our combination of Hotelling's model with tie-in sales. With tie-in sales the tying firm's profit function still exhibits a discontinuity. But the discontinuity is at a price that is not profit-maximizing. By contrast, the profit function of the tying firm's competitor exhibits no discontinuity. But the competitor cannot induce all consumers to give up the monopoly good with a small price reduction.

[2]Note that tie-in sales and bundling coincide in our model. Therefore, we use tie-in sales, tying, and bundling as synonyms.

The paper is organized as follows: In section II we set up the model. Next, we derive the demand functions and the equilibrium in section III. In section IV we conclude.

2 The Model

Consider two firms 1 and 2 and two markets A and B. Firm 1 is a monopolist in market A. It offers a non-differentiable good A. By contrast, firm 1 competes with firm 2 in market B. Both firms supply good B that is identical in all respects except one characteristic. A line with length one describes all possible values of this characteristic. The firms locate on this unit line. Let q_i, $i = 1, 2$, denote firm i's location. We assume that firm 1 cannot locate to the right of firm 2, i.e., $q_1 \leq q_2$. Unit and fixed costs are zero for both firms and both goods. We want to show and understand equilibrium existence in horizontal product differentiation with linear transportation costs and tie-in sales. Therefore, we focus on pure tying. Firm 1 only offers a bundle containing one unit of each good A and B.

There is a continuum of consumers with unit mass. Each consumer demands at most one unit of good A. The consumers have valuations r_A for A. Valuations r_A are uniformly distributed on the interval $[0, 1]$.

Each consumer has unit demand for good B. We denote by β a consumer's address on the unit line. This address reflects consumer β's most preferred location or good characteristic for good B. Consumers' addresses are uniformly distributed along the unit interval $[0, 1]$ with unit density. Let t be transportation costs per unit distance. Then a consumer incurs linear transportation costs $t|q - \beta|$ if her address differs from sales location q. Consumers' valuations for good B are high enough that each consumer buys a single unit of good B irrespective of its price. This assumption corresponds to full market coverage.[3]

In our model, full market coverage implies that consumers choose between two options: either they buy from firm 1 a bundle containing both products, or they do not buy good A at all and purchase only good B from firm 2. Irrespective of consumers' addresses firm 1 charges the mill price p_1 for the bundle. Likewise, firm 2 sells good B to all consumers at the same mill price p_2. Firms pass on total transportation costs to the consumers. Thus, consumers pay a full price consisting of the mill price and transportation costs.

Full coverage in market B has a second implication. We know that all consumers buy good B either in the bundle or separately. Since all consumers buy good B either way, gross valuation for B is irrelevant for consumers' buying decision. Or, put the other way around, the only relevant valuation for the buying decision is r_A. Therefore, the cases when valuation r_A and gross valuation for B are perfectly correlated, perfectly negatively correlated, and uncorrelated coincide in our model. Note, however, that r_A and address β are uncorrelated.

The set-up gives rise to the following two stage game: In the first stage, the firms simultaneously choose their locations. In the second stage, the firms simultaneously set prices. We look for a subgame perfect equilibrium in pure strategies.

[3]Hotelling-type models with partially covered markets are an interesting topic in itself, but lie outside the focus of this paper. For Hotelling-type models with partially covered markets we refer to the existing literature. See, e.g., Böckem (1994), Economides (1984), Hinloopen and van Marrewijk (1999), and Wang and Yang (1999)

3 The Equilibrium

3.1 Demand Specification

First of all, we need the demand functions to find the game's equilibrium. In our model it is possible that consumers buy from firm i although they have an address in firm j's hinterland. To see why, consider all consumers with $\beta \leq q_1$. These consumers buy firm 1's bundle if it yields a higher surplus than consuming only good B. Consumers willing to pay more than the difference between the price difference and the transportation costs difference travel to firm 1:

$$r_A \geq (p_1 - p_2) - t(q_2 - q_1). \tag{1}$$

Consumers buy the bundle if their valuations for A satisfy condition 1. But the valuations differ. Some consumers do not find A attractive enough to purchase the bundle. Hence, not all consumers to firm 1's left buy from firm 1. Analogously, some consumers with $\beta > q_2$ value A high enough that they buy the bundle.

The criterion given by condition 1 has a further implication. With respect to transportation costs, consumers with $\beta \leq q_1$ assess only the distance between q_1 and q_2. For a consumer living to the left of firm 1 transportation costs from covering the way to q_1 accrue anyway, independent of the address. Then, valuation r_A is the only variable that affects the buying decision. The analogous reasoning holds for all consumers with $\beta > q_2$.

To identify the demand functions we divide the unit line into three regions as shown in figure 1. Region X contains consumers with $\beta \leq q_1$. All consumers with $q_1 < \beta \leq q_2$ belong to region Y. In region Z lie all consumers

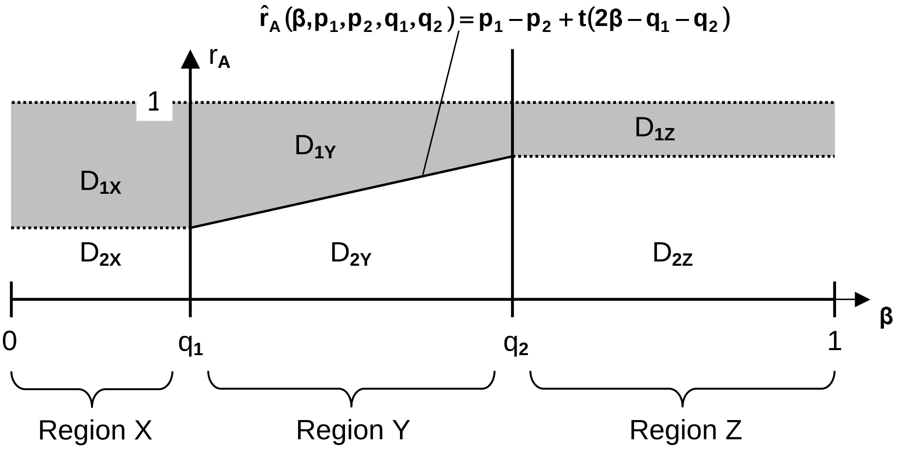

Figure 1: Demand Regions

to the right of firm 2's location, $q_2 < \beta$. Firm i serves demand D_{iR} in the respective regions $R = X, Y, Z$.

Demand Functions D_{iX} and D_{iZ}: For deriving the demand functions in region X and Z analogous arguments hold. So, allow us to derive only the demand function D_{iX} for region X in detail. In region X consumers' addresses are irrelevant as argued above. All consumers with valuations satisfying condition 1 buy the bundle. The indifferent consumers are given by the equality

$$r_A = p_1 - p_2 - t(q_2 - q_1). \tag{2}$$

Demand in region X only consists of consumers to firm 1's left. The demand functions in region X are

$$\begin{aligned} D_{1X} &= q_1 Prob\left[r_A \geq p_1 - p_2 - t(q_2 - q_1)\right] \\ &= q_1\left(1 - p_1 + p_2 + t(q_2 - q_1)\right), \end{aligned} \tag{3}$$

and

$$\begin{aligned} D_{2X} &= q_1 Prob\left[r_A < p_1 - p_2 - t(q_2 - q_1)\right] \\ &= q_1\left(p_1 - p_2 - t(q_2 - q_1)\right). \end{aligned} \tag{4}$$

Note that the probabilities in equations 3 and 4 must lie in the interval $[0, 1]$, at least in equilibrium. If the probabilities do not satisfy the condition to lie in $[0, 1]$, one firm serves the entire market. But this firm is not profit-maximzing. This firm can increase its price without losing consumers unless the probabilities lie in $[0, 1]$.

Analogous reasoning gives the demand functions for region Z:

$$D_{1Z} = (1 - q_2)(1 - p_1 + p_2 - t(q_2 - q_1)), \tag{5}$$

$$D_{2Z} = (1 - q_2)(p_1 - p_2 + t(q_2 - q_1)). \tag{6}$$

Demand $\boldsymbol{D_{iY}}$: Consumers with addresses in region Y base their buying decision on valuation r_A and the full prices. All consumers with net utilities

$$r_A - t(\beta - q_1) - p_1 \geq -t(q_2 - \beta) - p_2 \tag{7}$$

demand the bundle. Solving this decision rule for r_A yields the indifferent consumers' valuations depending on address, prices and locations:

$$\hat{r}_A(\beta, p_1, p_2, q_1, q_2) = p_1 - p_2 + t\left(2\beta - q_1 - q_2\right). \tag{8}$$

Figure 1 depicts the function $\hat{r}_A$ for the indifferent consumers' valuations. The function $\hat{r}_A$ gives for each address the minimal valuation a consumer must have, so that she buys the bundle. Thus, unlike in region X and Z, address β affects the buying decision. A consumer who buys the bundle and has an address far away from q_1 incurs high transportation costs whereas transportation costs are lower when buying from firm 2. The consumer only buys from 1 if consumption of A compensates for the higher transportation costs. As $\partial \hat{r}_A(\beta)/\partial\beta > 0$ shows, addresses closer to q_2 require a higher r_A. Summing up $\hat{r}_A$ over region Y results in the fraction of consumers that buy from firm 2. Hence, demand functions in region Y are:

$$D_{1Y} = q_2 - q_1 - \int_{q_1}^{q_2} \hat{r}_A(\beta)d\beta = (1 - p_1 + p_2)(q_2 - q_1), \tag{9}$$

$$D_{2Y} = \int_{q_1}^{q_2} \hat{r}_A(\beta)d\beta = (p_1 - p_2)(q_2 - q_1). \tag{10}$$

Figure 1 illustrates the demand for firm 1's bundle and firm 2's good. The shaded area represents all consumers who have β-r_A-combinations such that they buy the bundle. Firm 2 serves demand corresponding to the non-shaded area. Implicitly, we assume that both firms serve a fraction of consumers in every region $R = X, Y, Z$. This assumption turns out to be implied by existence of pure strategy equilibria.

Finally, we can state total demand for the bundle and for 2's good B. Summing up the demand in each region gives total demand D_i:

$$D_1 = D_{1X} + D_{1Y} + D_{1Z} = 1 - p_1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2), \tag{11}$$

$$D_2 = D_{2X} + D_{2Y} + D_{2Z} = p_1 - p_2 + t(q_2 - q_1)(1 - q_1 - q_2). \tag{12}$$

The demand functions exhibit an important characteristic for symmetric locations if prices are fixed. At fixed prices and for symmetric locations - that is, for $q_1 + q_2 = 1$ - demand is independent of locations and unit distance costs t. For an intuitive argument consider a situation with firms located some arbitrary distance away from each other. Furthermore, consider only symmetric locations. Prices are fixed. First, we look at the consumers to the left of firm 1 in region X. If firms symmetrically move closer to each other, firm 2's good B and the bundle are less differentiated. Since good B and the bundle are less differentiated, the difference in transportation costs decreases. With a smaller transportation cost difference, consumers with low valuation r_A switch from buying the bundle to buying firm 2's good B. Demand for the bundle decreases. For good B demand increases.

Exactly the same process occurs to the right of firm 2 in region Z, but with opposite sign. This demand change in region Z exactly outweighs the demand change in the region X. Total demand for the bundle as well as firm 2's good B does not change. Consequently, demand is independent of locations and per unit distance transportation costs for symmetric locations and fixed prices.

3.2 The Firms' Behavior

In the second stage firms set prices given locations and the opponent's price. The firms maximize profits

$$\pi_1 = p_1 D_1 = p_1 \left[1 - p_1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2)\right], \tag{13}$$

$$\pi_2 = p_2 D_2 = p_2 \left[p_1 - p_2 + t(q_2 - q_1)(1 - q_1 - q_2)\right], \tag{14}$$

with respect to their prices. Maximizing and solving firms' profits with respect to prices gives the firms' reaction functions:

$$p_1(p_2) = \left(1 + p_2 - t(q_2 - q_1)(1 - q_1 - q_2)\right)/2, \tag{15}$$

$$p_2(p_1) = \left(p_1 + t(q_2 - q_1)(1 - q_1 - q_2)\right)/2. \tag{16}$$

Both price reaction functions are linear in the other firm's price and are positively sloped. It follows that the reaction functions are well-behaved in the sense that they intersect only once. We can solve the system of equations given by the reaction functions to obtain optimal prices for the second stage as functions of locations:

$$p_1^*(q_1, q_2) = (2 - t(q_2 - q_1)(1 - q_1 - q_2))/3, \tag{17}$$

$$p_2^*(q_1, q_2) = (1 + t(q_2 - q_1)(1 - q_1 - q_2))/3. \tag{18}$$

Next, we turn to the first stage. Firms choose their profit-maximizing locations. Given their optimal pricing behavior, firms maximize profits

$$\pi_1 = [2 - t(q_2 - q_1)(1 - q_1 - q_2)]^2/9, \tag{19}$$

$$\pi_2 = [1 + t(q_2 - q_1)(1 - q_1 - q_2)]^2/9, \tag{20}$$

with respect to their locations. The firms' F.O.Cs. are

$$\partial\pi_1/\partial q_1 = 2tp_1^*(q_1, q_2)(1 - 2q_1)/3 = 0, \tag{21}$$

and

$$\partial\pi_2/\partial q_2 = 2tp_2^*(q_1, q_2)(1 - 2q_2)/3 = 0. \tag{22}$$

The firms' F.O.Cs. show that the firms locate at 1/2. Otherwise, the firms choose locations such that prices equal zero. If prices equal zero, the sufficient second order conditions for a global maximum fail. In other words, firms are not profit-maximizing if prices are zero. The following Proposition 1 summarizes the firms' equilibrium behavior:

Proposition 1 *In the Hotelling game with tie-in sales firms set equilibrium* *prices* $p_1^* = 2/3$ *and* $p_2^* = 1/3$*. Both firms locate at* $q = 1/2$*. Equilibrium profits are* $\pi_1^* = 4/9$ *and* $\pi_2^* = 1/9$*.*

In Hotelling's original model firms choose minimal differentiation. If product differentiation is minimal, a small price reduction attracts all consumers. Therefore, the firms undercut each other. This effect does not occur in our model. No firm lowers its price although they choose the same location. To get the entire market firm 1 needs to lower its price below firm 2's price. Firm 1's profits when serving all consumers are $\pi_1 = p_2 - t(q_2 - q_1) - \epsilon$. If both firms locate at $1/2$, firm 1 earns no more than $1/9 < \pi_1^*$. Hence, firm 1 does not change its price given its opponent's price.

If firm 2 wants to attract all consumers by a price reduction, it must compensate consumers for forgoing good A. Unlike in Hotelling's standard model firm 2 does not win all consumers if it lowers its price by a small amount ϵ. Because firms choose the same locations, their good B is homogenous. Then, consumers prefer firm 2's good over the bundle if $r_A - p_1 \leq -p_2$. In equilibrium, the indifferent consumer has the valuation $\hat{r}_A = 1/3$. If firm 2 lowers its price by ϵ the equation $r_A = p_1^* - p_2^* + \epsilon$ identifies the new indifferent consumer. We see that firm 2's price reduction by ϵ increases demand for its good to the same extent. In this case firm 2 earns profits $(1/3+\epsilon)(p_2^* - \epsilon) < \pi_2^*$. Hence, firm 2 does not change its price given firm 1's price.

Our analysis shows that tying reduces competition that otherwise prevails in the duopoly market because it differentiates firms' products. This differentiation resembles vertical product differentiation. Thereby, the monopolistic good serves as surrogate for, e.g., quality. It is the same competition-softening effect as described by Carbajo, de Meza, and Seidmann.

The competition-softening effect stems from the change in substitution relationships for goods. Recall that Martin observes a change in substitution between goods due to bundling. We find a similar effect in our model. Without bundling both firms' good B are homogeneous. With bundling firm 1 sells both goods A and B together. This bundle and good B are no longer homogeneous. Moreover, good A and good B become substitutes although they may be originally independent in demand. Thus, bundling changes the substitution relationships.

Let us relate firms' location choice in our model also to the principle of maximum differentiation introduced by d'Aspremont, Gabszewicz, and Thisse (1979). According to the principle of maximum differentiation firms locate at the endpoints of the unit line. Thus, in our model firms always choose less separation than in the model of d'Aspremont, Gabszewicz, and Thisse with quadratic transportation costs.

A comparison between our result and the principle of maximum differentiation raises a further question: Does our result depend on the specific transportation costs assumption? To deal with the question about transportation costs dependence we compare the outcomes of two versions for our model. We make a comparison between the outcome with linear transportation costs and the outcome with quadratic transportation costs. But we neither formally derive nor provide extensive proofs for the equilibrium outcome with quadratic transportation costs. A result based on the existing literature and intuition satisfies the requirement for a conjecture about transportation costs dependence.

For a result based on existing literature and intuition we draw on an analysis about quality and variety competition by Neven and Thisse (1990). Neven and Thisse challenge the generality of the principle of maximum differentiation in the case of quadratic transportation costs by adding a second dimension. This second dimension represents vertical product differentiation.

Recall that we can interpret product differentiation due to tying as vertical product differentiation. With this interpretation, we can apply the following result by Neven and Thisse for the case with fixed quality. If firms are already differentiated along the quality dimension, firms select a central location on the dimension for good characteristics. With maximum differentiation on the quality dimension, price competition is already soft. Instead of softening competition even more, firms prefer a central location on the line for good characteristics. Hence, our conjecture is that Proposition 1 still holds for quadratic transportation costs. We expect that our result in Proposition 1 does not depend on the specific assumption of linear or quadratic transportation costs.

4 Conclusions

In this paper we combine Hotelling's model of horizontal differentiation with tie-in sales to address equilibrium existence. We adopt a widely used setting: Firm 1 is a monopolist in some market A and faces competition by firm 2 in another market B. In our model Hotelling's principle of minimum differentiation holds: firms choose zero differentiation. But neither the bundling firm nor its competitor undercuts. The reason for equilibrium stability is a competition-softening effect due to tie-in sales. Tie-in sales themselves differentiate goods.

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