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Warranties, Moral Hazard and

the Lemons Problem

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Abstract

This paper analyses the following problem. Consumers cannot observe product quality at the time of purchase so that only warranties can induce firms to supply high quality products. Yet, if consumers can adjust the care with which they use products, the presence of warranties may result in more frequent product failure. The paper studies what kinds of contracts will be offered in a competitive market characterized by such a double sided moral hazard problem.

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I Introduction

This paper analyses the following problem. Consumers cannot observe product quality at the time of purchase so that only warranties can induce firms to supply high quality products. Yet, if consumers can adjust the care with which they use products, the presence of warranties may result in more frequent product failure. The paper studies what kinds of contracts will be offered in a competitive market characterized by such a double sided moral hazard problem.

If product quality cannot be observed by consumers at the time of purchase and reputations cannot be built, firms have no incentive to supply high quality products in the absence of warranties. Since 'lemons' can be produced at lower costs, they yield higher profits. Prosser [12] already noted that many manufacturers would provide consumers with 'worthless junk' without judicial intervention to imply 'warranties of quality'. Warranties are an obvious institution to counteract the 'lemons' problem because they penalize such behaviour on the part of producers. By lowering the quality level, firms increase the probability of product failure and thereby the costs of providing an additional unit of warranty. Accordingly, warranties which are large enough provide sellers with the incentive to supply high quality products (see Spence [15]).

Yet, the scope for warranties as an incentive device for firms to supply high quality products is restricted when consumers can adjust the care with which they use products (see McKean [8], Priest [11]). In the way they handle a product, consumers exert some influence on the probability of product failure. If the action chosen by consumers cannot be monitored, producers providing warranties face a moral hazard problem. The more warranty buyers get, the less incentive they have to avoid the event of product break-down.

Accordingly, the following trade-off emerges. As an incentive device for producers to supply high quality products, warranties have to be large enough. But the higher the warranty level is, the lower is the incentive for consumers to take care. Shapiro [13] even rules out warranties as a quality-assuring mechanism, due to the consumers' moral hazard problem. He claims that there is usually room for potential quality cutting by the seller, the warranty notwithstanding.

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Taking up this issue, this paper provides an exhaustive analysis of a market characterized by such a double sided moral hazard problem. Quality is taken to be the probability that the product works. Firms are risk neutral and can either provide a high or a low quality level. Consumers are risk averse. A consumer's effort is assumed to take two possible values. By the switch from high to low effort, consumers increase the failure probability of the product.

To analyse this double sided moral hazard problem, we consider the following two stage game. In the first stage, firms offer quality levels and price-warranty combinations. In the second stage, consumers observe the price-warranty combinations but not the quality levels. According to their beliefs consumers evaluate the expected utility of each offer available in the market and adjust their effort level optimally. The choice of effort cannot be monitored by firms. Consumers purchase the offer which generates the highest expected utility. Firms play Bertrand-Nash strategies in the first stage. We focus on the sequential equilibria of this game with imperfect information.

The analysis of consumer behaviour shows that there exist cheap warranties with a rationed indemnity. These incentive compatible warranties induce consumers to choose high effort in contrast to the complete but expensive warranties where they select low effort. We first study the case where consumers prefer the offer combining high quality with full insurance to the offer which specifies high quality and an incentive compatible warranty. This happens if the disutility of choosing high effort is large in relation to the gain from the decrease in the failure probability. In this case the market solves the 'lemons' problem. If at all, there are only efficiency losses due to the consumers' moral hazard problem. It is not possible to provide them with cheap and complete insurance.

We then consider the opposite situation where consumers prefer high quality with an incentive compatible warranty to high quality with full insurance. We describe the case where the offer which is most preferred by consumers induces with its corresponding incentive compatible warranty firms to produce high quality products. Again, efficiency losses arise only due to the consumers' moral hazard problem. We finally deal with the case where the offer

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which is most preferred by consumers does not specify an incentive compatible warranty which is large enough to induce high quality production. If there are some benefits from trade left, in equilibrium contracts will be traded which are worse for consumers than their most preferred one. Consumers only leave the market when the very last possible benefits from trade have disappeared.

A different but related approach by Cooper and Ross [3] also deals with the use of product warranties in a double moral hazard situation. In contrast to our model, they assume that consumers are risk neutral so that warranties do not serve for risk sharing purposes. They consider the following two stage game. In the first stage, a single firm and a single consumer determine cooperatively a price-warranty combination. In the second noncooperative stage players act simultaneously. The firm decides which quality to produce and the consumer selects his optimal effort level. Cooper and Ross assume that a stable and unique Nash-equilibrium for the second stage of this game exists. Under this assumption they characterize the equilibrium choices of quality, care and the warranty level. Whereas Cooper and Ross only focus on the role of warranties as an incentive device, we also study their effect on the allocation of risks.

Section II of the paper describes the model. The players' pay-off functions are derived in section III. In section IV the existence results are established. In section V we present some comparative statics results. The welfare properties of the equilibria are discussed in section VI.

II The Model

a) Consumers

Consider a one-commodity market with a large set of identical consumers who have initial wealth M. Consumers are interested in purchasing a single unit of the good in question. The commodity may either work satisfactorily or break down completely. The probability that the product works depends on i) its quality and ii) the level of care e with which consumers handle the product.

Let $p \geq 0$ be the price charged and let $w \geq 0$ be a monetary warranty. Then the consumers' von Neumann-Morgenstern utility will be

$$\tilde{V} = \begin{cases} U(M - p + 1) - \alpha e, & \text{if the good works,} \\ U(M - p + w) - \alpha e, & \text{if the good does not work,} \end{cases}$$

where $U(\cdot)$ is a function from $I\!R_+$ into $I\!R$ with $U'(\cdot) > 0$ and $U''(\cdot) < 0$. The parameter $\alpha > 0$ measures the disutility of effort. For simplicity we assume that the consumer's utility function is additively separable in income and effort. In this case, the consumer's preferences over income lotteries are independent of effort.

Consumers are assumed to have two choices of effort, i.e. $e \in \{0, 1\}$. The product has the working probability q when consumers choose the low effort level $e = 0$. The consumer increases the working probability to $q + \epsilon$ by switching to the high effort level $e = 1$. For a fixed quality, the consumer's choice of effort finally determines the probability of product failure. In the sequel we will identify a good by its minimum working probability, or equivalently quality, q. We will assume that quality levels $q \in [0, 1 - \epsilon]$ are potentially available in the market.

A consumer purchases a product of quality q and a contract $\gamma = (p, w)$ specifying a price p and a warranty w. A consumer's expected utility is given by

$$V(q, \gamma, e) = (q + \epsilon e)U(M - p + 1) + (1 - q - \epsilon e)U(M - p + w) - \alpha e.$$

Obviously, we are only interested in situations where the disutility of effort is less than the additional utility of having a working product, meaning

$$0 < \alpha < U(M+1) - U(M).$$

Our concern is with moral hazard problems. This means that the provision of warranties affects the level of care chosen by consumers. Therefore, we have to ensure that the consumer chooses the high effort level $e = 1$ if he purchases a product of quality q with a contract $\bar{\gamma} = (p, 0)$ which specifies no warranty, i.e.

$$V(q, \bar{\gamma}, 1) \geq V(q, \bar{\gamma}, 0) \quad \text{or}$$

$$\epsilon \geq \alpha / \big(U(M - p + 1) - U(M - p)\big).$$

We ensure that in the absence of any warranty the consumer chooses high effort by assuming

$$\epsilon \geq \alpha / \big(U(M+1) - U(M)\big). \tag{1}$$

Under this assumption the expected loss from the switch to $e = 0$ is large enough in relation to the disutility of effort to induce consumers to provide $e = 1$ in the absence of any warranty.

b) Firms

The production sector is assumed to consist of N identical firms. Firms can produce two quality levels, i.e. $q \in \{q_L, q_H\}$ with $1 - \epsilon \geq q_H > q_L \geq 0$. Following Mussa and Rosen [9], a firm can produce any number of products of quality q at a unit cost $C(q)$. For the sake of simplicity, we further assume constant marginal quality costs, meaning $C(q) = c \cdot (q + \epsilon)$ where $c \in (0, 1)$ is chosen so that $V(q_H, c, 0, 1) > U(M)$. The assumption of constant marginal quality costs is by no means essential. What is actually needed, is a unique optimal quality level for consumers if the product were sold at unit costs $C(q)$. In this model the optimal quality is, due to the linearity of the cost function, the high quality level q_H, i.e. $V\big(q_H, C(q_H), 0, 1\big) > V\big(q_L, C(q_L), 0, 1\big)$. The restriction on the range of c ensures that there is already potential surplus in the market if the high quality level q_H is offered without any warranty.

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c) Game Structure and Equilibrium Concept

In the first stage of the game, each firm $i \in \{1, .., N\}$ selects a strategy specifying a quality level $q_i \in \{q_L, q_H\}$ and a contract $\gamma_i \in I\!R_+^2$. In the second stage, consumers observe the contract of each firm but not its respective quality level. Consumers attempt to infer the quality level $q_i \in \{q_L, q_H\}$ of firm $i \in \{1, .., N\}$ from the observed values $(\gamma_i)_{i=1}^N \equiv \Gamma$. According to their beliefs consumers evaluate the expected utility of each contract γ_i and adjust their effort level $e \in \{0, 1\}$ optimally. The choice of effort cannot be monitored by firms. Among all contracts consumers then either choose the one generating the highest expected surplus if this turns out to be nonnegative. Otherwise, they leave the market. Given the contracts of all other firms, the consumers' behaviour at the second stage defines each firm's profit. By the choice of a quality level and a contract each firm maximizes its expected profits.

For this two stage game with imperfect information we adopt the sequential equilibrium concept as developed by Kreps and Wilson [7]. At the second stage, consumers observe the contracts Γ of all N firms. For all possible values of Γ, consumers have beliefs which denote the probabilities that firm $i \in \{1, .., N\}$ produces high resp. low quality. Consumers select the optimal effort level for each contract according to their beliefs. Then they choose the one generating the highest expected utility or they leave the market. Given the consumers' strategies, firms play Bertrand-Nash equilibrium stragegies in the first stage of the game.

Each decision maker acts in a sequentially rational fashion, following a strategy from each point forward that maximizes his expected payoff given his current information and beliefs. In equilibrium the consumers' beliefs are borne out: what consumers expect is what firms actually choose to do. In our problem it will turn out that the firms' strategic quality choice is completely determined by the choice of a contract, independent of what consumers actually believe and of the effort level they provide. We restrict the consumers' beliefs to reflect this dominant quality choice by firms.

III Derivation of the Pay-off Functions

In this section we will derive the pay-off functions for each player. We will first analyse the consumers' effort choice problem in case they know the actual quality a firm produces. We will then study the firms' strategic choice of quality. We will finally close the whole setup by the definition of the consumers' beliefs.

At the second stage of the game, consumers observe the price-warranty combinations Γ which firms offer. Given a belief system, these contracts completely determine the consumers' behaviour. Therefore, we carry out the following analysis in the contract space. Without loss of generality, we will consider the situation where warranty payments cannot exceed service capacity, i.e. $w \leq 1$. If consumers get full insurance $w = 1$, the maximum price they are willing to pay equals unity for all possible quality levels. Accordingly, we confine our attention to the set of contracts $\{\gamma | 0 \leq p, w \leq 1\}$.

Let us first analyse the consumers' strategic choice of effort. A consumer compares whether he is better off by choosing $e = 1$ (and thereby $q + \epsilon$) or low effort $e = 0$ (and thereby q). Consider a consumer who purchases a product of quality q with a contract γ. If the contract specifies no warranty, i.e. $w = 0$, by assumption (1) the consumer provides high effort. If the consumer gets the complete warranty $w = 1$, he does not care about the break-down probability because the utility in case the product works equals the utility in case the product fails. To avoid the disutility of effort α, the consumer chooses low effort in this case. Accordingly, by continuity we can deduce that there exists a warranty level where the consumer is indifferent with respect to the choice of effort.

First note that this warranty level, denoted by $\phi(p) \in (0, 1)$, is independent of q. $\phi(p)$ is defined by

$$V\big(q, p, \phi(p), 1\big) - V\big(q, p, \phi(p), 0\big) =$$

$$\epsilon[U\big(M - p + 1\big) - U\big(M - p + \phi(p)\big)] - \alpha = 0 \qquad \text{or}$$

$$\phi(p) = U^{-1}\big(U(M - p + 1) - \alpha/\epsilon\big) - M + p \in (0, 1) \quad \forall p \in [0, 1]. \tag{2}$$

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By the switch to low effort the consumer gains α. He loses ϵ times the difference in utility between a working and a failing product. At the warranty level $\phi(p)$ this loss in expected utility outweighs the reduced effort cost. The warranty level $\phi(p)$ is independent of q since for all possible quality levels $q \in [0, 1 - \epsilon]$ the switch to $e = 1$ increases the working probability by ϵ and the effort cost by α. By inspection of (2) we find that $\phi(p)$ is unique. Differentiating (2) further yields $\phi'(p) \in (0, 1)$.

Accordingly, whenever a contract specifies a warranty level $w < \phi(p)$, the consumer chooses high effort. If $w > \phi(p)$, the consumer chooses low effort. We adopt the convention that for warranty levels $w = \phi(p)$ where the consumer is indifferent with respect to the choice of effort, he provides high effort. Define $E(\gamma) = 1$ if $w \leq \phi(p)$ and $E(\gamma) = 0$ if $w > \phi(p)$ to be the optimal choice of effort given γ. The preceeding observations are summarized by figures 1-3.

Next, let us analyse the firms' quality choice problem. A strategy of a firm is a contract and a choice of a quality level $q \in \{q_L, q_H\}$. If risk neutral firms offer a product of quality q with a contract γ the expected profit per consumer equals in case the contract is purchased

$$\pi(q, \gamma, e) = \begin{cases} p - c \cdot (q + \epsilon) - (1 - q - \epsilon)w, & \text{for } e = 1, \\ p - c \cdot (q + \epsilon) - (1 - q)w, & \text{for } e = 0. \end{cases}$$

To study the firms' strategic quality choice, consider the iso-profit lines for the two quality levels q_L, q_H which a firm can produce. See figures 1-3. For both choices of the consumers' effort the two iso-profit lines of producing high resp. low quality intersect in $w = c$. For warranty levels $w < c$, a firm makes higher profits by producing q_L instead of q_H and vice versa. This argument is independent of what consumers actually believe and of the effort level they choose. It is thus a dominant strategy for firms to produce q_H if $w > c$ and q_L if $w < c$. We adopt the convention that at $w = c$ where the firm is indifferent, it produces q_H. A strategy of a firm is a contract and a choice of q. But if firms are acting optimally, a contract γ implies the strategic quality choice of the firm (except for the case $w = c$). Therefore, define $Q(w) = q_L$ if $w < c$ and $Q(w) = q_H$ if $w \geq c$ to be the optimal quality of firms given w.

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Let us now define the consumers' beliefs $\mu = (\mu_1, .., \mu_N)$ where μ_i denotes the probability that firm i produces q_H. We restrict beliefs to reflect the implications of the dominant quality choice by firms. Accordingly, when consumers observe a warranty level $w_i \geq c$, then with probability 1 they believe that firm i produces q_H, regardless of the contracts of the other firms. For $0 \leq w_i < c$ the corresponding probability is 0.

Given their beliefs, consumers select the optimal effort level for each contract γ in the market. Denote the consumers' expected utility from purchasing a contract γ when the firm chooses its optimal level of q and the consumer his optimal level of e by

$$W(\gamma) = V\big(Q(w), \gamma, E(\gamma)\big).$$

Consumers then purchase the contract generating the highest expected surplus, provided this is nonnegative; otherwise, they leave the market. Firm $i's$ expected profit per consumer when it chooses its optimal level of q and consumers choose their optimal level of e and purchase their best contract is given by

$$\pi_i\big(\gamma_i, \Gamma\big) = h_i(\Gamma)\big[p_i - C\big(Q(w_i)\big) - \big(1 - Q(w_i) - \epsilon E(\gamma_i)\big)w_i\big]$$

where

$$h_i(\Gamma) = \begin{cases} 1, & \text{if } W(\gamma_i) \geq W(\gamma_j) \geq U(M) \\ & \text{for all } j \in \{1, .., N\} \\ 0, & \text{if } W(\gamma_i) < W(\gamma_j) \\ & \text{for some } j \in \{1, .., N\} \\ & \text{or } W(\gamma_i) < U(M). \end{cases}$$

We adopt the convention that each firm has some customers when consumers face the same contract from several firms. Firms act as Bertrand-Nash competitors. That is, each firm i takes the behaviour of consumers and the contracts of all other firms as fixed and chooses a contract γ_i so as to maximize expected profits.

IV Equilibrium Results

To characterize the equilibrium outcomes let us forget for a moment about the double moral hazard probem and analyse the role of warranties as a mere insurance device. Consider the case where firms offer the warranty at the respective fair odds rate $(1 - q_i - \epsilon e), q_i \in \{q_L, q_H\}, e \in \{0, 1\}$. Then we have that for all warranty levels $w < 1$, risk averse consumers are only partially insured. They appropriate additional surplus by raising the insurance up to the complete warranty $w = 1$.

Next, let us further take into account the consumers' moral hazard problem. Denote the warranty levels which are defined by the intersections of the respective zero-profit lines of $\pi(q_i, \gamma, e)$ with the graph of $w = \phi(p)$ by $w(q_i, e)$, $q_i \in \{q_L, q_H\}, e \in \{0, 1\}$. Consider the offer $(q_H, \hat{\gamma})$ where $\hat{\gamma} = \big(C(q_H) + (1 - q_H - \epsilon)w(q_H, 1), w(q_H, 1)\big)$. This offer combines the quality level q_H with the highest incentive compatible warranty $w(q_H, 1)$ at the fair odds rate $(1 - q_H - \epsilon)$. It generates a positive expected surplus, i.e. $V(q_H, \hat{\gamma}, 1) > U(M)$. We have assumed that there is already potential surplus in the market when the high quality level q_H is offered without any warranty at marginal quality costs $C(q_H)$. By our preceeding analysis about the consumers' strategic choice of effort we know that $w(q_H, 1) \in (0, 1)$. Since $w(q_H, 1)$ is sold at the fair odds rate, consumers appropriate additional surplus by being partially insured. The offer $(q_H, \hat{\gamma})$ will turn out as a useful point of reference in characterizing the equlibrium outcomes.

Let us now return to our double moral hazard problem. Divide the set of contracts into four regions, depending on the value of E and Q.

(1) $E(\gamma) = 0$ and $Q(w) = q_H$

(2) $E(\gamma) = 0$ and $Q(w) = q_L$

(3) $E(\gamma) = 1$ and $Q(w) = q_H$

(4) $E(\gamma) = 1$ and $Q(w) = q_L$

Let γ^i be the utility maximizing contract in region (i) subject to the nonnegative profit condition. These contracts will be the equilibrium candidates for our two stage game.

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Consider first region (1). Firms produce high quality if $w \geq c$. Consumers provide low effort whenever $w > \phi(p)$. See figures 1-3. Firms break even in this low effort - high quality region if they offer contracts on the zero-profit lines of $\pi(q_H, \gamma, 0)$ and $w \in \left(\max[w(q_H, 0), c], 1 \right]$. This interval is always nonempty. If firms offer the insurance at the fair odds rate $(1 - q_H)$, consumers choose the complete warranty $w = 1$. Hence, the utility maximizing low effort - high quality contract γ^1 is given as $\left(C(q_H) + (1 - q_H), 1 \right)$.

Next consider region (2). Firms produce low quality if $w < c$ and consumers provide low effort if $w > \phi(p)$. If $w(q_L, 0) > c$, firms make losses when they offer a contract in this low effort - low quality region. See figures 1-2. If $w(q_L, 0) \leq c$, firms break even when they offer contracts on the zero-profit line of $\pi(q_L, \gamma, 0)$ and $w \in \left(w(q_L, 0), c \right)$. If firms offer insurance at the fair odds rate $(1 - q_L)$ out of this interval, we have that $V \left(q_L, C(q_L) + (1 - q_L)w, w, 0 \right) < V \left(q_L, C(q_L) + (1 - q_L), 1, 0 \right)$. Both contracts offer insurance at the fair odds rate $(1 - q_L)$. The second contract specifies the complete warranty whereas contracts out of region (2) only partially insure consumers. Next note that $V \left(q_L, C(q_L) + (1 - q_L), 1, 0 \right) < W(\gamma^1)$ as $q_L < q_H$. Hence, we can conclude that consumers always prefer the low effort - high quality contract γ^1 to any contract out of region (2) yielding nonnegative profits.

Consider now region (3). Consumers provide high effort if $w \leq \phi(p)$. Firms produce high quality if $w \geq c$. If $w(q_H, 1) \geq c$, firms break even in this high effort - high quality region if they offer contracts on the zero-profit line of $\pi(q_H, \gamma, 1)$ and $w \in [c, w(q_H, 1)]$. We then obviously have that the utility maximizing high effort - high quality contract γ^3 equals $\hat{\gamma}$, specifying the highest incentive compatible warranty. See figure 1. If $w(q_H, 1) < c$ we have that $Q \left(w(q_H, 1) \right) = q_L$. To induce firms to produce q_H, the warranty level has to be greater or equal c. To induce consumers to provide high effort, utility maximizing contracts have to lie on the curve $w = \phi(p)$. Along the line $\phi(p)$ we calculate that

$$\partial V \left(q, p, \phi(p), 1 \right) / \partial p = (1 - q - \epsilon)\left(\phi'(p) - 1 \right) U'\left(M - p + \phi(p) \right) - (q + \epsilon)U'(M - p + 1)$$

which is negative as $\phi'(p) \in (0, 1)$. A movement down the line $\phi(p)$ increases the consumers' expected utility since the price decreases by more than the corresponding warranty level. The-

refore, the consumers' expected utility is maximized by the contract $\gamma^3 = \left(\phi^{-1}(c), c\right)$ specifying the lowest warranty level which induces firms to produce q_H. See figures 2-3. Note that $\gamma^3 = \left(\phi^{-1}(c), c\right)$ entails positive profits for firms since it lies above the zero-profit line of $\pi(q_H, \gamma, 1)$.

Consider now region (4). Firms produce low quality if $w < c$ and consumers provide high effort if $w \leq \phi(p)$. If $w(q_L, 1) < c$, firms break even in this high effort - low quality region if they offer contracts on the zero-profit line of $\pi(q_L, \gamma, 1)$ and $w \in [0, w(q_L, 1)]$. The utility maximizing high effort - low quality contract is then given as $\gamma^4 = \left(C(q_L) + (1 - q_L - \epsilon)w(q_L, 1), w(q_L, 1)\right)$. See figures 2-3. If $w(q_L, 1) \geq c$, firms make zero profits high effort whenever $w \in [0, c)$. See figure 1. In this case, any break-even contract out of region (4) is dominanted by the high effort - high quality contract γ^3. The two zero-profit lines of $\pi(q_i, \gamma, 1)$, $q_i \in \{q_L, q_H\}$ intersect in (c, c). $w(q_L, 1) \geq c$ implies that $\phi^{-1}\left(w(q_L, 1)\right) \geq c$. Since $\phi'(p) \in (0, 1)$, we have that $\phi^{-1}(c) \leq c$. The zero-profit line of $\pi(q_H, \gamma, 1)$ lies between the zero-profit line of $\pi(q_L, \gamma, 1)$ and the graph of $p = c$ for $w \geq c$. By the intermediate value theorem we can therefore conclude that $w(q_H, 1) \geq c$. This in turn implies that for $w(q_L, 1) \geq c$, $\gamma^3 = \hat{\gamma}$. We then obviously have that $W(\gamma^3) > V\left(q_L, C(q_L) + (1 - q_L - \epsilon)w, w, 1\right)$ $\forall w \in [0, c)$. Accordingly, $w(q_L, 1) \geq c$ implies $\gamma^3 = \hat{\gamma}$ and in this case consumers prefer the high effort - high quality contract γ^3 to all contracts out of region (4) yielding nonnegative profits. An analogous argument shows that $w(q_H, 1) < c$ implies $w(q_L, 1) < c$.

We may now establish the existence results. First, let us examine the case where the disutility of effort α is so large and/or the gain from the decrease in the failure probability ϵ so small, that consumers prefer the low effort - high quality contract γ^1 specifying full insurance to the refernce offer $(q_H, \hat{\gamma})$ which combines high quality with the incentive compatible warranty $w(q_H, 1)$.

Proposition 1: *Given c, α, ϵ, q_H satisfy $W(\gamma^1) > V(q_H, \hat{\gamma}, 1)$. Then, in a sequential equilibrium all firms offer the contract γ^1 and produce q_H. Consumers choose $e = 0$ and all firms*

have customers.

<u>Proof:</u> When consumers observe all firms offering γ^1, because $w = 1 > c$ they know that each firm produces q_H. Furthermore, the optimal choice of effort is $e = 0$. All firms offer the same contract. Accordingly, each firm has customers. The firms' expected profit per consumer is zero.

We still have to check whether a firm can increase its profits by unilaterally deviating from the equilibrium strategies. Consider first the contracts out of the low effort regions (1) and (2). The low effort - high quality contract γ^1 is the utility maximizing one in this set subject to the nonnegative profit condition. Any contract more favourable to consumers would incur losses; any contract with less favourable terms would have no customers. Accordingly, no firm has an incentive to change γ^1. A firm which maintains the contract γ^1 and switches to q_L would incur losses. Hence, no firm wishes to deviate by a strategy which induces consumers to choose low effort.

A firm might consider to increase its profits by offering a contract out of the high effort regions (3) and (4). If consumers could discern the respective quality level, the strategy $(q_H, \hat{\gamma})$ would be the best offer for consumers which is incentive compatible and does not yield losses. But according to the presumption we have that $W(\gamma^1) > V(q_H, \hat{\gamma}, 1)$. Hence, a firm offering any such incentive compatible contract would have no customers.

$\square$

Let us now analyse the opposite case where consumers prefer the reference offer $(q_H, \hat{\gamma})$ to the low effort - high quality contract γ^1 specifying full insurance. Now the outcome depends on the relation of marginal quality costs c to the warranty level $w(q_H, 1)$.

insert: **Figure 1: The Case of Proposition 2**

Proposition 2: *Suppose $V(q_H, \hat{\gamma}, 1) \geq W(\gamma^1)$ and $c \in \big(0, w(q_H, 1)\big]$ Then, in a sequential equilibrium all firms offer the contract $\gamma^3 = \hat{\gamma}$ and produce q_H. Consumers choose $e = 1$ and all firms have customers.*

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<u>Proof:</u> If $w(q_H, 1) \geq c$, consumers know that firms produce q_H. The optimal choice of effort is $e = 1$. All firms offer the same contract. Accordingly, each producer has a clientele. The firms' expected profit per consumer is zero.

The contract $\gamma^3 = \hat{\gamma}$ specifies the best terms for consumers within the high effort regions (3) and (4) subject to the nonnegative profit condition. Therefore, a firm has no incentive to change the price and/or the warranty level. A firm which maintains γ^3 and switches to the low quality level q_L cannot increase its profits. Hence, no firm wishes to deviate by an incentive compatible strategy.

A firm might consider to increase its profits by offering a contract out of the low effort regions (1) and (2). Among these contracts, γ^1 is the utility maximizing one for consumers. But according to the presumption we have that $W(\gamma^3) \geq W(\gamma^1)$. Hence, a firm offering a contract out of region (1) or (2) would have no customers.

$\square$

Let us now consider the final situation where marginal quality costs c exceed $w(q_H, 1)$. In this case we have $\gamma^1 = \left(\phi^{-1}(c), c\right)$ and, since $w(q_H, 1) < c$ implies $w(q_L, 1) < c$, that $\gamma^4 = \left(C(q_L) + (1 - q_L - \epsilon)w(q_L, 1), w(q_L, 1)\right)$. Let $\gamma^0 = (0, 0)$ be the no trade contract with $W(\gamma^0) = U(M)$.

insert: **Figure 2: The Case of Proposition 3 i)**

insert: **Figure 3: The Case of Proposition 3 ii) -3 iv)**

Proposition 3: *Suppose $V(q_H, \hat{\gamma}, 1) > W(\gamma^3)$ and $c > w(q_H, 1)$.*

i) *If $W(\gamma^3) \geq W(\gamma^0), W(\gamma^1), W(\gamma^4)$, in a sequential equilibrium all firms offer the contract γ^3 and produce q_H. Consumers choose $e = 1$ and all firms have customers.*

ii) *If $W(\gamma^4) \geq W(\gamma^0), W(\gamma^1), W(\gamma^3)$, in a sequential equilibrium all firms offer the contract γ^4 and produce q_L. Consumers choose $e = 1$ and all firms have customers.*

iii) *If $W(\gamma^1) \geq W(\gamma^0), W(\gamma^3), W(\gamma^4)$, in a sequential equilibrium all firms offer the contract*

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γ^1 and produce q_H. Consumers choose $e = 0$ and all firms have customers.

iv) If $W(\gamma^1), W(\gamma^3), W(\gamma^4) < W(\gamma^0)$, in a sequential equilibrium firms produce nothing and offer the contract γ^0. Consumers leave the market and no firm has customers.

Proof: i) When consumers observe $\gamma^3 = (\phi^{-1}(c), c)$, they know that firms produce q_H. The optimal choice of effort is $e = 1$. γ^3 generates a nonnegative expected surplus. Hence, all firms have customers and make positive expected profits. See figure 2.

A firm might try to increase its profits by deviating with another incentive compatible contract. For those incentive compatible contracts with $w < c$, consumers know that they face the low quality level q_L. γ^4 is the utility maximizing contract for consumers out of region (4). A firm offering γ^4 would have no customers because $W(\gamma^3) \geq W(\gamma^4)$. A firm which offers γ^3 has no incentive to switch to q_L. At the warranty level c firms make the same expected profit for both possible quality levels. Hence, no firm wishes to deviate with another incentive compatible strategy.

Among the feasible contracts which induce consumers to choose low effort, γ^1 is the utility maximizing one. A firm offering γ^1 would have no customers as $W(\gamma^3) \geq W(\gamma^1)$. Accordingly, no firm can increase its profits by offering a contract which induces consumers to choose $e = 0$.

ii), iii) The proofs are similar to the ones already given and are omitted. See figure 3.

iv) Neither the low effort - high quality contract γ^1 nor the high effort - low quality contract γ^4 generate a nonnegative expected surplus. The only candidates for a trade equilibrium remain contracts which combine the high quality level q_H with an incentive compatible warranty. The offer $(q_H, \hat{\gamma})$ would generate a positive expected surplus if consumers knew the actual quality level q_H. But with $w(q_H, 1) < c$, consumers know that they face the low quality level q_L. Among the contracts out of region (3), $\gamma^3 = (\phi^{-1}(c), c)$ is the best one for consumers. But γ^3 generates a negative expected surplus. Hence, consumers leave the market while firms produce nothing and offer the no trade contract γ^0.

□

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V Comparative Statics Results

Let us now perform some comparative statics exercises with respect to the severity of the moral hazard problem. First note that increasing the disutility of effort directly lowers the expected utility of any incentive compatible contract because α enters the utility function $V(\cdot)$ negatively. Differentiating (2) gives us $\partial\phi(p)/\partial\alpha < 0$. An increase in the disutility of effort shifts the curve of indifference with respect to the choice of effort $\phi^{-1}(w)$ to the left. With a higher disutility of effort, consumers prefer to switch to $e = 0$ at lower warranty levels. This shift of the curve $\phi^{-1}(w)$ lowers the incentive compatible warranty $w(q_H, 1)$, meaning less insurance at the fair odds rate. Obviously, $V(q_H, \hat{\gamma}, 1)$ decreases with α. The low effort - high quality contract γ^1 and the expected utility it generates are independent of α. Therefore, we may conclude that raising α makes the case of Proposition 1 where $W(\gamma^1) > V(q_H, \hat{\gamma}, 1)$ more likely to occur.

Suppose now we are in the situation of Proposition 2 where the high effort - high quality contract $\gamma^3 = \hat{\gamma}$ is traded and let $V(q_H, \hat{\gamma}, 1) > W(\gamma^1)$. Then a small increase in α lowers the incentive compatible warranty $w(q_H, 1)$. If $w(q_H, 1)$ becomes smaller than marginal quality costs c, we end up in the case of Proposition 3 where the market fails to provide consumers with their most preferred offer. In this situation we may therefore conclude that the more severe is the consumers' moral hazard problem, the lower the cost of quality must be to provide consumers with their most preferred contract $\hat{\gamma}$.

In the situation of Proposition 3 i), the induced shift of $\phi^{-1}(w)$ increases the price $\phi^{-1}(c)$ consumers have to pay for the high effort - high quality, positive profits contract γ^3. This effect lowers $W(\gamma^3)$ and makes cases iii) and iv) more likely to occur. Again, marginal quality costs c have to fall to remain at this equilibrium. In case ii), the shift of $\phi^{-1}(w)$ lowers $w(q_L, 1)$. Accordingly, the expected utility of the high effort - low quality contract γ^4 falls with α, making cases iii) and iv) more likely. In cases iii) and iv), we have that the low effort - high quality contract γ^1 and the no trade contract γ^0 as well as $W(\gamma^1)$ and $W(\gamma^0)$ are independent of the disutility of effort. Note further that $\partial\phi(p)/\partial\epsilon > 0$. Lowering the increase in the failure

probability ϵ also shifts the curve $\phi^{-1}(w)$ to the left. Accordingly, the entire argumentation with respect to increasing α also applies with respect to decreasing ϵ.

Let us conclude this section with some comparative statics results about the equilibrium warranty and quality levels. Consider the case of Proposition 2 where the high effort - high quality contract $\gamma^3 = \hat{\gamma}$ is traded. If $w(q_H, 1) > c$, the incentive compatible warranty $w(q_H, 1)$ decreases with q_H. The decrease in $w(q_H, 1)$ has a negative effect on the consumers' expected utility. Yet, this negative effect is more than offset by the effect of the higher quality and the lower price, making consumers overall better off. Accordingly, when the high effort - high quality contract $\gamma^3 = \hat{\gamma}$ is traded we observe a negative correlation between quality and warranty which coincides with empirical observations. See Cooper and Ross [3] and Priest [11]. In the situation of Proposition 3 ii), we have that $w(q_L, 1)$ increases with q_L. Thus, when the high effort - low quality contract γ^4 is traded, we observe a positive correlation between quality and warranty.

VI Welfare Analysis

Before we can analyse the welfare properties of the equilibria, we have to define an ordering over different outcomes. As *first-best* we define the outcome, a planner can achieve who is able to determine the quality produced by firms and the effort chosen by consumers. We will call the outcome *second-best* if the planner can determine the quality produced by firms but not the level of care provided by customers. The alternative possibility where there is no monitoring problem concerning the choice of effort but a 'lemons' problem, is not interesting as a point of reference because it yields the first-best outcome. The high quality level q_H is traded with full insurance while effort is adjusted in a socially optimal way.

The welfare properties of the equilibrium of Proposition 1 where firms produce high quality and consumers supply no effort are as follows. Consider the offer (q_H, γ^*) where $\gamma^* = \left(C(q_H) + (1 - q_H - \epsilon), 1 \right)$ provides high quality with full insurance at the high effort fair odds rate. Obviously, γ^* is not incentive compatible. If $W(\gamma^1) > V(q_H, \hat{\gamma}, 1)$ implies $W(\gamma^1) > V(q_H, \gamma^*, 1)$, then the equilibrium is first best. This is clearly the case for q_H close enough to $(1 - \epsilon)$. If

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$W(\gamma^1) < V(q_H, \gamma^*, 1)$, then the equilibrium is second-best. Individual rationality destroys the social optimum (q_H, γ^*). See e.g. Pauly [10] for a more detailed discussion of this typical moral hazard phenomenon.

The equilibrium of Proposition 2 where firms produce high quality and consumers provide full effort has the following welfare properties. For $q_H = (1 - \epsilon)$ consumers do not care about the level of insurance and the equilibrium is first-best. For $q_H < (1 - \epsilon)$ the equilibrium is second-best, i.e. $W(\gamma^3) < V(q_H, \gamma^*, 1)$. Consumers lose potential surplus because they only get the incentive compatible warranty $w(q_H, 1) < 1$. The 'lemons' problem is solved by the market in an efficient way.

In the case of Proposition 3 the moral hazard effects are so severe that the contract $\hat{\gamma}$ which is the best incentive compatible one for consumers, no longer induces high quality production. A planner who is able to determine the quality level, can make consumers better off. When the high effort - high quality contract $\gamma^3 = (\phi^{-1}(c), c)$ is traded in equilibrium, firms make positive profits although they play Bertrand-Nash strategies. This result is an implication of the two effort choice and the dominant quality choice by firms which is reflected in the consumers' belief system. See Arnott and Stiglitz [1] and Hellwig [4] for a more detailed discussion of positive profits equilibria in one sided moral hazard situations. Although in the case of Proposition 3 the market fails to provide consumers with their most preferred incentive compatible offer $(q_H, \hat{\gamma})$, consumers only leave the market when there are no more benefits from trade left.

The reason for this market failure is the fact that if the product fails, the compensation awarded to the consumer and the penalty paid by the producer are the same. Consider the situation where the producer pays a fine large enough to induce high quality production. The consumer obtains an incentive compatible compensation. The difference between the two payments is given to a neutral third party. The market could then provide consumers with their most preferred incentive compatible offer. This observation was first pointed out by Shavell [14].

This separation of the compensation given to the consumer and the penalty paid by the

producer raises the problem of credibility. If the producer's commitment to pay an additional fine to a neutral third party in case of product failure is not enforceable, he will refuse to pay ex post. Hence, the commitment is not credible. But even an enforceable commitment need not be credible as both consumer and producer have an incentive to form a coalition. If the product fails, the producer can avoid the payment to the third party by giving the consumer more than the ex ante agreed upon compensation. In return the consumer does not reveal the fact of product break-down to the third party. By such an agreement in the spirit of Coase [2], consumer and producer can be made better off. Anticipating this behaviour, we are back in the situation where the fine is equal to the compensation.

VI Conclusions

We have provided an exhaustive characterization of a market where warranties are used to solve a double sided moral hazard problem. Due to the fact that risk and warranties are tied to the product, we do not encounter the difficulties of pure insurance markets with moral hazard. In insurance models risk is inherent to consumers. If consumers are offered incentive compatible rationed insurance policies, they try to purchase several policies from different insurers to gain at least a complete indemnity. This kind of the consumers' behaviour makes the rationing of the amount of indemnity a difficult matter in these markets. See e.g. Hellwig [4], [5] and Pauly [10].

Yet, if insurance firms offer a supplementary warranty at the respective low effort premium-benefit ratio, producers can no longer offer contracts which specify an incentive compatible warranty. The rationed warranty combined with the supplementary one is no longer incentive compatible. This observation was made by Jaynes [6] for insurance markets with adverse selection.

We have found that out of six possible equilibrium constellations, only in two warranties do not solve the 'lemons' problem because of the consumers' moral hazard problem. Shapiro's [13] claim that warranties do not serve as a quality-assuring mechanism is only valid for the

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constellations of Proposition 3 ii) and 3 iv). It remains of course an empirical problem to assess the relevance of the respective constellations.

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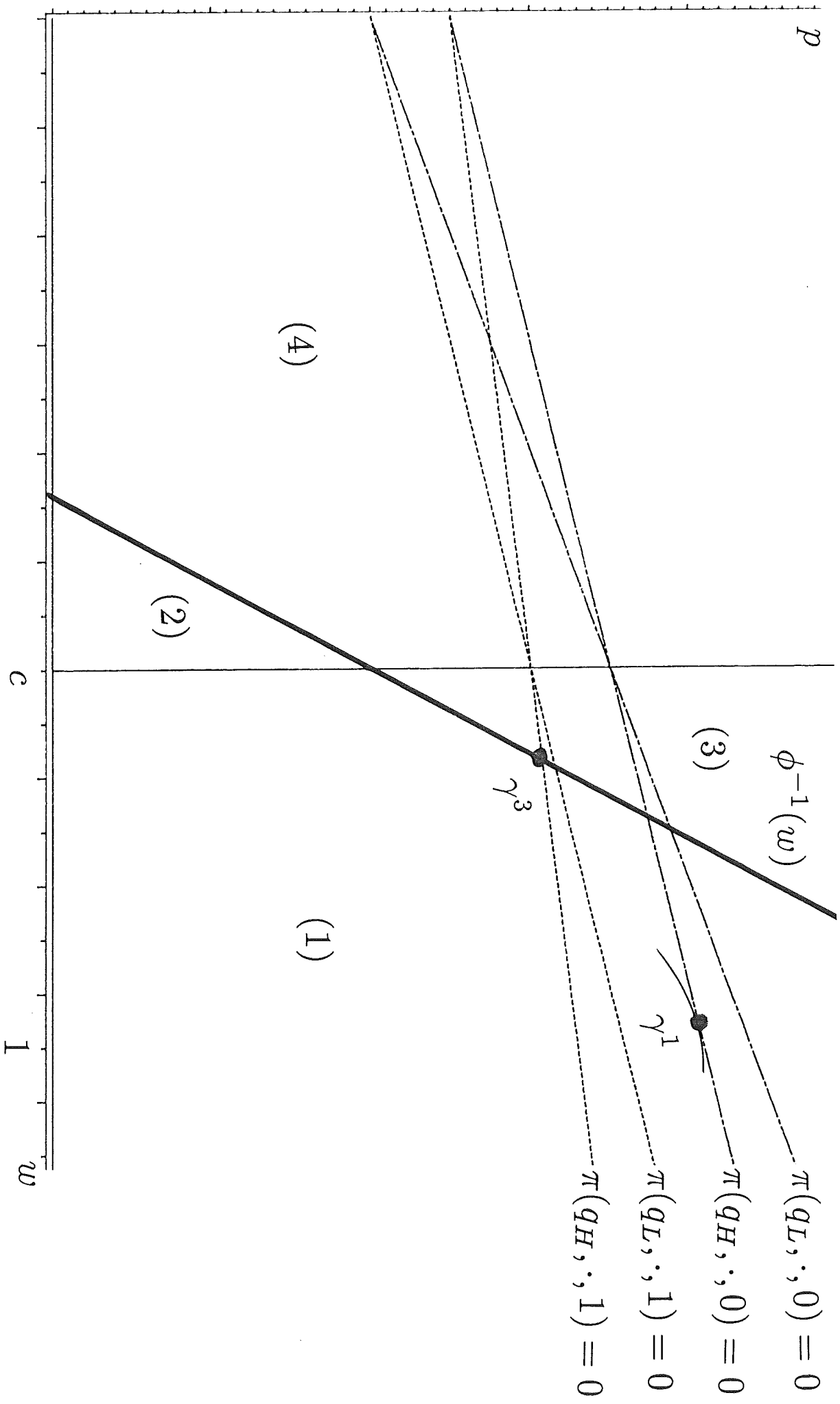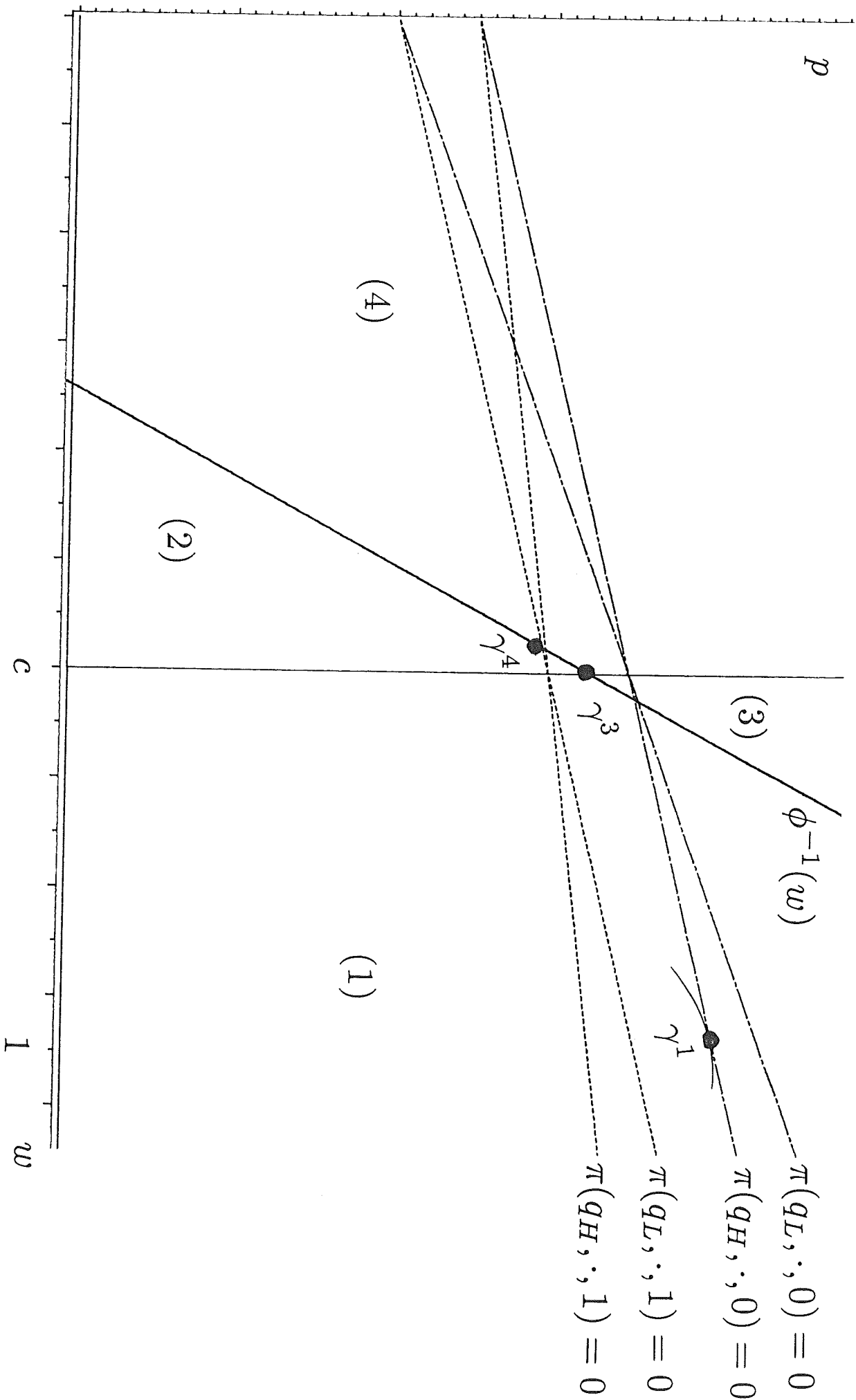Figure 1: The case of Proposition 2

Figure 2: The case of Proposition 3 i)

Figure 3: The case of Proposition 3 ii) - 3 iv)