Adversarial versus Inquisitorial Testimony [*]

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Version: July 2019

Abstract

An arbiter has to decide a case under a purely adversarial procedure. He can do so using his priors, or the two parties to the conflict may present further evidence. The parties can distort the evidence in their favor at a cost. In equilibrium the two parties never testify together. When the evidence is much in favor of one party, this party testifies; the testimony is, however, distorted. When the evidence is close to the prior mean, no party testifies. We compare this result with the outcome under a purely inquisitorial procedure where the arbiter decides how much testimony he wants to hear.

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Comparaison des témoignages entre les procédures accusatoire et inquisitoire

Résumé

Un arbitre doit décider d'un litige entre deux parties dans le cadre
d'une procédure purement accusatoire. Sa décision peut reposer uni-
quement sur ses a priori, en l'absence d'information supplémentaire ;
ou bien elle peut reposer sur les éléments de preuve additionnels sou-
mis volontairement par les parties au litige. Lorsqu'elle décide de
témoigner, et moyennant un coût, chaque partie peut déformer la
réalité en sa faveur. À l'équilibre, les parties ne témoignent jamais si-
multanément. Une partie ne témoigne que si les faits sont suffisamment
en sa faveur, mais son témoignage présente alors une version exagérée
de cet avantage. Inversement, si les faits sont proches de l'évaluation
a priori du décideur, aucune des parties ne témoigne. Nous comparons
cet équilibre à celui qui serait obtenu avec une procédure purement
inquisitoire où c'est l'arbitre qui décide du nombre de témoignages.

1 Introduction

An arbiter has to decide a dispute between two parties. The underlying facts of the case are in question. The arbiter can decide the case solely on the basis of his priors, implying that the verdict is most likely not accurate. Alternatively, the parties to the conflict may present further evidence. They will, however, try to spend resources to distort the evidence in their favor. This means that higher accuracy is achieved at the expense of costly falsification.

Two different institutions have emerged to deal with this trade-off: the inquisitorial and the adversarial procedure. Under the *inquisitorial procedure* the arbiter decides how much testimony he wants to hear. We have studied the inquisitorial procedure in Emons and Fluet (2009).[1] In the paper at hand we analyze the adversarial procedure. Under the *adversarial procedure* it is for the parties to decide whether they testify or not.[2] We derive the equilibrium testifying behavior and analyze the welfare properties. Moreover, we compare these results with our earlier ones to determine under which conditions one procedure does better than the other.

The two parties have diametrically opposed interests about the issue on which the arbiter has to decide. For example, in a divorce case the issue may be the amount of support she owes to him, or in a breach of contract case the damages the defendant owes to the plaintiff. Both parties know the actual amount of support/damages and both would like to influence the arbiter's decision.

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1. Under the inquisitorial system "it is for the judge to examine the witnesses, if any, it is for the judge to decide whether to summon the parties for interrogation and it is the judge who acts to obtain the assistance of an expert when required," Jolowicz (2000, p. 220). In Emons and Fluet (2013) we analyze the inquisitorial procedure in a setting where the adjudicator commits to linear adjudication rules.

2. Under the adversary system "it is for the parties to determine not only the issues which the court is to decide, but also the material on which the decision will be based. The evidence presented to the court will be that which the parties choose to present and none other. The judge may not require that a particular witness be summoned to give evidence or that a particular document be produced; he may not even question the witnesses himself except for the purpose of clarifying some doubt as to the meaning of what a witness has said under examination by counsel," Jolowicz (2000, p. 28).

As a benchmark we first look at a pure disclosure framework. Parties can only submit hard information, thereby disclosing the true value. Presenting evidence involves a fixed cost. Alternatively, the parties may remain silent.

In a purely adversarial procedure the parties decide whether or not to present testimony. The arbiter is passive at the discovery stage. Once the parties have finished, he decides the case on the basis of his priors and of what can be inferred from the parties' actions. The arbiter seeks to minimize adjudication error, implying that his sequentially rational decision is to adjudicate the posterior mean. When he hears no testimony the posterior mean equals the prior, which the arbiter adjudicates. When he hears testimony, he knows and adjudicates the true value.

The pure disclosure game has a unique equilibrium. The defendant testifies for low values of damages, the plaintiff for high values, and for values in between both parties remain silent. We measure welfare by summing the social loss from inaccurate adjudication and the parties' submission costs. The equilibrium has the following virtues. When actual damages are close to the prior mean so that the informational value is small, the parties remain silent. There are no submission costs, yet some inaccuracy. Only when the informational value is high, do the parties come forward and testify.

Under a purely inquisitorial procedure the arbiter decides how much testimony he wants to hear. He summons no, one, or both parties. With no testimony submission costs are zero and the arbiter adjudicates the mean, leading to a loss from inaccurate decisions. In the pure disclosure set-up the arbiter will never ask both parties to testify: one testimony reveals the truth. If one party testifies, the arbiter adjudicates the true value, yet at the expense of the submission cost. Under the inquisitorial procedure there is no fine-tuning as to the realization of damages; but the arbiter has full control over which kind of costs he incurs.

Under the adversarial procedure the arbiter always expects both, silence and testimony. Under the inquisitorial procedure he fully controls the amount of testimony. When he cares little about accurate decisions, he refuses to hear

the parties; when he cares a lot, he asks a party to testify. Therefore, the inquisitorial procedure does better than the adversarial one when the arbiter cares little or a lot about accurate decisions. The adversarial procedure, by contrast, does better when the arbiter cares to a similar extent about both, accurate decisions and submission costs.

Next we allow parties to inflate testimony. The parties can boost the evidence in either direction. Distorting the evidence involves, however, an additional cost: the cost goes up with the misrepresentation at an increasing rate. For instance, it becomes increasingly more difficult to produce the fake documents the further they are away from the truth. We now have a signalling game.

The equilibrium under the adversarial procedure is similar to the pure disclosure equilibrium. No party testifies when the true value is close to the prior mean and thus influencing the arbiter has negligible private value. When, however, the evidence is sufficiently in favor of one party, this party comes forward and testifies.

But now parties inflate their testimony. If the plaintiff testifies, he overstates the true value; if the defendant testifies, he understates the true value. Boasting increases the more the true amount differs from the prior mean. The arbiter rationally anticipates the inflation and adjudicates the true value. Accordingly, when parties testify, the equilibrium is revealing but it involves falsification. Since parties are silent when the true value is close to the prior mean and inflate when the true value is sufficiently in favor of one party, the equilibrium involves both falsification costs and error costs.

Under the inquisitorial procedure when the arbiter hears one party, depending on who testifies, the party overstates or understates the true value. The arbiter rationally corrects for the exaggerated amount and adjudicates the true value. Accordingly, the equilibrium is revealing but it involves falsification.

When both parties have to submit, both testimonies involve falsification, one party over-reports while the other under-reports. The arbiter corrects for

this by taking an average of the exaggerated testimonies. Interestingly, under joint testimony a party inflates less than if he is the only one to testify: under joint testimony the arbiter attaches less weight to his claims than under single testimony. It turns out that total falsification costs are lower than under single testimony: two slightly distorted reports are less costly than a single heavily distorted one. Yet, fixed submission costs are duplicated. If fixed submission costs are low, joint testimony is cheaper than single testimony.

The welfare comparison yields the following results. When wasteful influence expenditures are not too large, the inquisitorial procedure performs better when the arbiter has strong views about error costs; otherwise the adversarial procedure does better. This result resembles our pure disclosure result.

Nevertheless, when falsification expenditures are large and fixed submission costs low, the inquisitorial procedure does better irrespective of the weight attached to accurate decisions: the inquisitorial arbiter requires joint submission leading to lower falsification expenditures than the adversarial procedure. In this case the inquisitorial procedure thus dominates the adversarial procedure.

We use legal procedures as an example. The same issues arise in regulatory or administrative proceedings, wage arbitration, as well as in many other contexts: a member of parliament makes a case for his constituency, and not for the others; directors of plants or heads of divisions compete for budgets or transfer prices; the minister for the environment cares about the environment, the minister for economic affairs promotes growth, and the prime minister arbitrates in case of conflict. [3]

Let us now turn to the literature. Accuracy and cost are generally considered the two most important criteria in comparing legal systems; see, e.g., Posner (1999). Supporters of the adversarial system claim that delegation of

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3. Milgrom (1988) argues that those who know much about the consequences of alternative decisions are also often affected a lot by these decisions. Therefore, bringing forth useful information comes at the cost of the wasteful influence activities of those who inform decision-makers.

evidence production to the parties ensures an accurate ascertainment of the facts. Advocates of the inquisitorial system view the adversarial system as prone to manipulation, leading to over-production and duplication of deceptive testimony; see Tullock (1975).

One body of the litigation literature has modeled the trial outcome using contest functions as a reduced-form approach to the arbiter's behavior. These functions generate the winning probability of each litigant depending on their litigation expenses. [4] Parties engage in a rent-seeking game, leading to excessive expenditures. Whereas in this literature the contest function is exogenously given, in our set-up the Bayesian arbiter takes into account the parties' incentives to submit misleading testimony.

Another strand of the litigation literature models trials as persuasion games. If a party testifies, it discloses the truth; it cannot falsify as such. The party can, however, remain silent. [5] As benchmark we also discuss the case of hard information. In our main framework, by contrast, the parties cannot disclose hard information; they dissipate resources in attempting to fabricate convincing stories.

In yet another literature the neutral inquisitorial arbiter searches for evidence whereas under the adversarial procedure the parties to the conflict control the uncovering of the evidence. [6] However, in civil litigation and by contrast with criminal trials, the presentation of evidence essentially rests with the parties even in so-called inquisitorial systems. The main difference is the judge's involvement in controlling the litigants' presentation of evidence through bench requests, questions, and the like. [7] Demougin and Fluet

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4. See Cooter and Rubinfeld (1989), Farmer and Pecorino (1999), Katz (1988), and Parisi (2002).

5. See Sobel (1985), Milgrom and Roberts (1986), Lewis and Poitevin (1997), and Shin (1998). A variant of this literature includes models where the parties engage in strategic sequential search of favorable evidence; see Froeb and Kobayashi (1996) and Daughety and Reinganum (2000).

6. See Shin (1998), Dewatripont and Tirole (1999), Froeb and Kobayashi (2001), and Palumbo (2001).

7. See Jolowicz (2000), Langbein (1985), or Parisi (2002) for a comparative description, along these lines, of adversarial and inquisitorial systems.

(2008) present an analysis of active versus passive judging in a persuasion game. They show that a more active or inquisitorial arbiter may eliminate inefficient equilibria. However, they do not deal with influence costs as such nor with the trade-off between submission costs and accuracy.

We model evidence presentation as a signalling game. The so-called costly state falsification approach generates costly testimonies as signals.[8] If one party testifies, we have a one-sender signalling game; testimony of both parties yields a two-sender game with perfectly correlated types.

Kartik et al. (2007) and Kartik (2009) consider a single sender who can send inflated messages. Boosting is costly. In the second paper the sender's type space is bounded. The sender communicates with a single receiver. In the first paper the sender's type space is unbounded and he communicates with one or more receivers. In both papers the sender has to send a message; he cannot be silent. The set-up is thus reminiscent of the inquisitorial procedure when one party has to testify.

In Emons and Fluet (2019) we look at a pure disclosure set-up with reporting costs. The state space is bounded and we allow for arbitrary (e.g. asymmetric) priors. We model the adversarial procedure as in the paper at hand, i.e., both parties are allowed to report. The major difference is in how we approach the inquisitorial procedure. In the paper at hand the arbiter bars, say, the plaintiff and summons the defendant, which means the defendant has to testify. There the arbiter only bars the plaintiff, the defendant is free to testify. The inquisitorial adjudicator may thus end up with no testimony at all. The welfare comparison is based solely on error costs.

The paper is organized as follows. In the next section we start with a set-up where the parties cannot falsify. In section 3 we extend the framework to inflated testimonies. Section 4 concludes. The detailed derivations of the welfare results are relegated to Appendix 1. All proofs are collected in

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8. Lacker and Weinberg (1989), Maggi and Rodríguez-Clare (1995), and Crocker and Morgan (1998) use this approach in a principal-agent context.

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Appendix 2.

2 Pure Disclosure

A plaintiff P has sued a defendant D. The issue which the arbiter has to decide is the amount of damages $x \in \mathbb{R}$. The plaintiff wants damages to be large while the defendant wants them to be small. The evidence available so far about x is given by the normal distribution with mean μ and variance σ^2. We denote the density by $f(x)$ and the cumulative by $F(x)$.[9] At the beginning of the trial all parties involved, i.e., plaintiff, defendant, and arbiter, know the distribution of x. The mean is such that, given the expected outcome at trial, the plaintiff's claim has positive net value.[10]

Once the trial has started, both plaintiff and defendant learn the realization of x; the fact that they have become perfectly informed is common knowledge. The trial cannot be stopped at this point; the adjudicator has to decide the case. In particular, we rule out any out-of-court settlement negotiations. The arbiter can adjudicate solely on the basis of his priors $f(x)$. Alternatively, he may receive further evidence submitted by the perfectly informed but self-interested plaintiff and defendant.

After plaintiff and defendant have become informed, they may testify. Testimony is costly. A submission is of the form "the value of the quantity at issue is x_i," $i = P, D$. In the pure disclosure set-up the parties can only submit hard information, thereby disclosing the true value, i.e., claims are restricted to $x_i = x$. Alternatively, the parties may refrain from testifying, which is denoted by $\emptyset_i$, $i = P, D$. A party's action is therefore $s_i \in \{\emptyset, x\}$. The cost is

$$c_i(s_i, x) = \begin{cases} \gamma, & \text{if } s_i = x; \\ 0, & \text{if } s_i = \emptyset, \end{cases}$$

9. With an unbounded state space we avoid boundary conditions. The probability of extreme, say, negative values of x can be made arbitrarily small.

10. We make this precise at the end of this section.

$i = P, D$ where $\gamma > 0$. The total cost of testimony is $C = c_P + c_D$. The arbiter observes the defendant's and the plaintiff's actions and then adjudicates $\hat{x}(s_P, s_D) \in \mathbb{R}$.

Society trades-off the loss from inaccurate adjudication l and the cost of testimony C, i.e., the social loss is

$$L = l + C.$$

Let $\hat{x}$ be the arbiter's decision. The loss from inaccurate adjudication or error cost is

$$l(\hat{x}, x) = \theta(\hat{x} - x)^2$$

where $\theta > 0$ is the weight society gives to error costs. Error costs are zero when the true value is adjudicated. The higher the error, the higher are error costs, increasing at an increasing rate the further one moves away from the truth. [11]

2.1   Adversarial procedure

Under a purely adversarial procedure, it is for the parties to decide whether they testify or not. The procedure is as follows. The parties observe x and simultaneously pick s_P and s_D. The arbiter observes the parties' actions and adjudicates $\hat{x}$. The arbiter adjudicates so as to minimize expected error costs.

The parties choose s_P and s_D so as to maximize π_P and π_D where

$$\pi_P(\hat{x}, s_P, x) = \hat{x} - c_P(s_P, x) \quad \text{and}$$
$$\pi_D(\hat{x}, s_D, x) = -\hat{x} - c_D(s_D, x).$$

After the arbiter has observed the agents' choices, he updates his beliefs; these are given by the probability distribution over x in the information set given by the parties' actions.

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11. Society wants correct judicial decisions so that legal rules provide the intended incentives; the regulator and the CEO want correct prices to avoid misallocations of resources.

The pure disclosure game has a straightforward unique equilibrium. The defendant testifies for low values of x, the plaintiff for high values, and for values in between both parties remain silent.

Proposition 1: *In the unique equilibrium of the adversarial procedure the plaintiff discloses when $x \geq \mu + \gamma$ and is silent otherwise. The defendant discloses when $x \leq \mu - \gamma$ and is silent otherwise. The judge adjudicates the true x when he hears testimony; otherwise, he adjudicates $\hat{x} = \mu$.*

The social loss is the sum of the error cost over the interval where the parties remain silent and the submission cost over the range of x where they testify. At equilibrium

$$L^A(\theta, \gamma) = \theta \int_{\mu-\gamma}^{\mu+\gamma} (x - \mu)^2 f(x)\, dx + [F(\mu - \gamma) + 1 - F(\mu + \gamma)]\gamma, \quad (1)$$

where the superscript A stands for the adversarial procedure.

We express the social loss as a function of the submission cost γ and the weight θ given to the error cost because we will perform our welfare analysis in this space. We immediately obtain $L^A(\theta, 0) := \lim_{\gamma \to 0} L^A(\theta, \gamma) = 0$. A reduction in γ not only reduces the cost of testimony but also the interval where parties are silent, thus also error costs. We also have $L^A(\theta, \infty) := \lim_{\gamma \to \infty} L^A(\theta, \gamma) = \theta\sigma^2$. For very high γ it becomes extremely unlikely to hear testimony at all. [12]

2.2 Inquisitorial procedure

Let us now turn to a purely inquisitorial procedure. Under this procedure the arbiter decides how much testimony he wants to hear. He does so as to minimize the total social loss $L = l + C$. Specifically, the arbiter first announces whether he wants to hear no, one, or both parties. Afterwards the arbiter adjudicates.

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12. $F(\mu - \gamma)$ and $1 - F(\mu + \gamma)$ decrease exponentially fast when γ goes to infinity. Hence, the second term in (1) goes to zero.

When the arbiter refuses testimony, submission costs are zero. The arbiter minimizes expected error costs solely on the basis of the priors, implying $\hat{x} = \mu$. The expected total loss is $L^I = \theta\sigma^2$ where the superscript I indicates the inquisitorial procedure. In the pure disclosure set-up the arbiter will never ask both parties to testify. One testimony reveals the truth and a second testimony only adds to submission costs. Therefore, the inquisitorial judge will order at most one party to talk, leading to the loss $L^I = \gamma$. The arbiter chooses the action leading to the smallest social loss, thus

$$L^I(\theta, \gamma) = \min\{\gamma, \theta\sigma^2\}. \tag{2}$$

2.3 Welfare comparison

From a welfare point of view the adversarial procedure has the following virtues.[13] When the social value of information is small (i.e., x is close to the prior μ), the private benefit of testifying is also small; the parties remain silent and do not spend resources on testifying, yet at the expense of some inaccuracy in adjudication. When the social value of information is large, the private benefit of disclosing is also large; the parties testify thus enabling correct decisions, yet at the expense of the cost of disclosing. Nevertheless, the parties' incentives to testify need not be perfectly aligned with the social value of information. By contrast, the inquisitorial procedure is all-or-nothing: ex ante it either enforces or forbids testimony; there is no fine-tuning.

The inquisitorial procedure does better when either (i) it forbids testimony, there is too much disclosure under the adversarial regime, and the arbiter does not care too much about correct decisions, or (ii) when it enforces testimony, there is too little disclosure under the adversarial system, and the arbiter cares a lot about correct decisions. Otherwise the adversarial procedure does a better job than the inquisitorial one.

Let us finally look at the plaintiff's decision to sue. Under the adversarial procedure, the plaintiff's expected payoff at equilibrium is $\mu - \gamma(1 - F(\mu +$

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13. Recall that formal statements on welfare comparison are collected in Appendix 1.

γ)). Under the inquisitorial procedure, his expected payoff is $\mu - \gamma$ if he thinks he will be required to testify; otherwise it is μ. The plaintiff sues if his expected payoff is positive. Under either procedure, $\mu > \gamma$ is clearly a sufficient condition.

3 Inflated Testimony

Let us now extend our pure disclosure model to a set-up where the parties can falsify the evidence at a cost. Again a submission is of the form "the value of the quantity at issue is x_i," $i = P, D$. The message is a claim or story conveyed in such a way that x_i looks real. The cost of a presentation is $\gamma + \kappa (x_i - x)^2$ where $\kappa > 0$. The actual value is x, which is observed by the party, and x_i is the testimony or the statement submitted.

Distorting testimony is costly, the more so the further one moves away from the truth. The cost of misrepresentation goes up at an increasing rate: for example, expert witnesses charge more the more they distort the truth, the so called "hired gun phenomenon." Falsification costs increase with κ; for κ going to zero falsification becomes costless and for κ arbitrarily large our pure disclosure set-up applies. [14]

A party's action is now $s_i \in \{\emptyset \cup \mathbb{R}\}$ with cost

$$c_i(s_i, x) = \begin{cases} \gamma + \kappa (x_i - x)^2, & \text{if } s_i = x_i \in \mathbb{R}; \\ 0, & \text{if } s_i = \emptyset, \end{cases} \tag{3}$$

$i = P, D$. We now have a signalling game. It differs from the usual signalling model in that two senders share the same information.

The message x_i in (3) is simply a costly action with the interpretation "the true state is x_i." The plaintiff tells the truth if $x_P = x$; he boosts his case, that is, he "lies" or "falsifies" in his favor if $x_P > x$. At equilibrium, of

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14. Maggi and Rodríguez-Clare (1995) use the same function and interpret κ as capturing the publicness of information. If $\kappa = 0$, information is purely private. For arbitrarily large κ, the public-information model obtains.

course, the arbiter may not believe a message and may take into account a party's incentive to make inflated claims.

3.1   Adversarial procedure

In principle we can have equilibria where testimony provides some information or we can have totally uninformative pooling equilibria. The latter possibility is easily discarded. First, standard refinements such as Grossman-Perry (1986) rule out equilibria where both parties would always remain silent. [15] Second, it cannot be the case that both parties always testify with claims that are invariant to the true state. [16]

We focus on revealing equilibria; by revealing we mean that the arbiter infers the true state when the parties testify. We impose the following structure.

No-understatement: At equilibrium, if P testifies at x, he claims $x_P \geq x$; if D testifies at x, he claims $x_D \leq x$.

Monotonicity: At equilibrium, if P testifies at x, he also testifies at $x' > x$ and $x_P(x') \geq x_P(x)$; if D testifies at x, he also testifies at $x'' < x$ and $x_D(x'') \leq x_D(x)$.

Minimality: At an out-of-equilibrium information set the arbiter believes that it was reached with the minimum number of deviations from the equilibrium strategies. [17]

To clarify minimality, consider an out-of-equilibrium pair (s_P, s_D). Suppose s_P is never observed at equilibrium but s_D is; then the arbiter believes

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15. The proof is in Appendix 2.

16. Suppose that P always claims x_P and D always claims x_D. The judge adjudicates $\hat{x} = \mu$. If P deviates to $\emptyset_P$, he adjudicates some ν. But then P's payoff from the deviation is $\nu > \mu - \gamma - \kappa(x_P - x)^2$ when x differs sufficiently from x_P. The same argument holds if D were to always play $\emptyset_D$.

17. A similar restriction on beliefs has been used by Bagwell and Ramey (1991), Schultz (1999), or Emons and Fluet (2009, 2012); see also Hetzendorf and Overgaard (2001) and Fluet and Garella (2001). These papers also involve two-sender signalling games with perfectly correlated information.

that the defendant played his equilibrium strategy while the plaintiff got it wrong. If both s_P and s_D are observed at equilibrium, although never simultaneously, the arbiter infers that (only) one party deviated but may not be sure which one. Finally, if neither s_P nor s_D is ever observed at equilibrium, minimality imposes no restriction on beliefs.

Lemma 1: *In a revealing equilibrium, (i) each party's strategy involves both silence and testimony; (ii) the plaintiff and the defendant never testify together.*

The Lemma implies that there exists $x_D^0 < x_P^0$ such that the defendant testifies when $x \leq x_D^0$ and is otherwise silent, while the plaintiff testifies when $x \geq x_P^0$ and is otherwise silent. The consequence is that, for, say, $x \geq x_P^0$, the plaintiff's equilibrium separating strategy $x_P(x)$ can be derived using the well-known methods of one-sender signalling games. [18]

For $x \geq x_P^0$, the arbiter's strategy is $\hat{x}(x_P, \emptyset_D)$ where x_P is the plaintiff's testimony. The plaintiff chooses x_P to maximize $\hat{x}(x_P, \emptyset_D) - c_P(x_P, x)$. If $x_P(x)$ is separating, the function is one-to-one and the arbiter therefore adjudicates $\hat{x}(x_P, \emptyset_D) = x_P^{-1}(x_P)$. Because the strategy is optimal for the plaintiff, he chooses x_P to maximize $x_P^{-1}(x_P) - c_P(x_P, x)$. The first-order condition to the plaintiff's problem yields the differential equation

$$(x_P(x) - x)\, x_P'(x) = \frac{1}{2\kappa}, \quad x \geq x_P^0. \tag{4}$$

We solve this equation using the non-decreasing solution and given the initial condition $x_P(x_P^0) = x_P^0$; the latter characterizes the least-cost signalling strategy, the so-called Riley equilibrium. Likewise, the defendant's strategy solves

$$(x - x_D(x))\, x_D'(x) = \frac{1}{2\kappa}, \quad x \leq x_D^0, \tag{5}$$

with $x_D(x_D^0) = x_D^0$. [19]

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18. See Mailath (1987) for signalling games with a continuum of types. The only difference is that the equilibrium profile $(x_P(x), \emptyset_D)$ needs to be supported by out-of-equilibrium beliefs preventing a deviation by D.

19. The solutions satisfy the conditions for a global maximum to the parties' optimization problem. See Mailath (1987).

The thresholds at which the parties decide to testify are a property of the equilibrium. We have that $x_D^0 = \mu - \gamma$ and $x_P^0 = \mu + \gamma$. Thus, the parties' decision whether or not to testify is the same as in the pure disclosure framework. Solving the differential equations yields:

(i) The plaintiff's testimony $x_P(x)$ is $x_P \geq x$ satisfying

$$x = x_P - \frac{1 - e^{-2\kappa(x_P - \mu - \gamma)}}{2\kappa}, \quad x \geq \mu + \gamma. \tag{6}$$

(ii) The defendant's testimony $x_D(x)$ is $x_D \leq x$ satisfying

$$x = x_D + \frac{1 - e^{-2\kappa(\mu - \gamma - x_D)}}{2\kappa}, \quad x \leq \mu - \gamma. \tag{7}$$

To sum up:

Proposition 2: *Under the adversarial procedure with least-cost separating strategies, the defendant claims x_D solving (7) when $x \leq \mu - \gamma$, while the plaintiff is silent. When $x \in (\mu - \gamma, \mu + \gamma)$, neither party testifies. When $x \geq \mu + \gamma$, the plaintiff claims x_P solving (6) and the defendant is silent. If one party testifies, the arbiter infers and adjudicates the true x; if neither party testifies, the arbiter rationally expects and adjudicates μ.*

The strategies are represented in Figure 1. At the threshold where he decides to testify, the plaintiff claims the truth. For $x > \mu + \gamma$, he inflates his claim, i.e., $x_P(x) > x$ with $\lim_{x \to \infty} x_P(x) = x + 1/(2\kappa)$. Boasting increases with the true state, yet at a decreasing rate. The defendant's strategy is similar. For $x < \mu - \gamma$, $x_D(x) < x$ with $\lim_{x \to -\infty} x_P(x) = x - 1/(2\kappa)$.

3.2 Inquisitorial procedure

Let us now summarize our results from Emons and Fluet (2009). When the arbiter requires the plaintiff to testify, the separating strategy $x_P(x)$ will also solve the differential equation (4) but without the boundary condition; that is, the equation must hold for all $x \in \mathbb{R}$. Similarly, when the defendant is required to testify, $x_D(x)$ will solve the differential equation (5) over the whole real line.
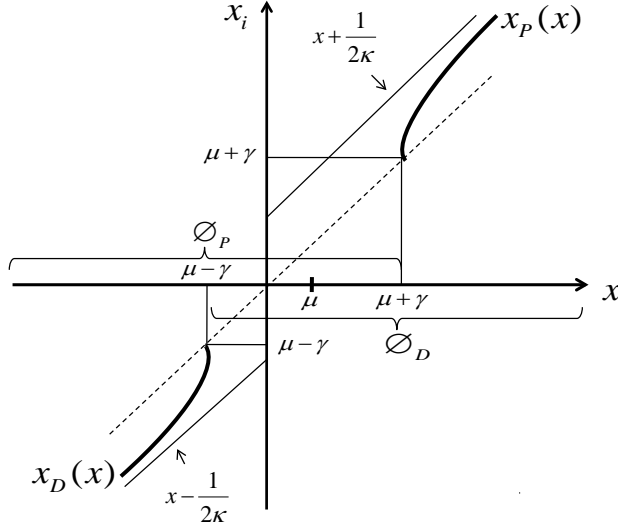
Figure 1: The least cost signalling strategies $x_P(x)$ and $\mathrm{x}_D(x)$

The least-cost separating strategies, i.e., the ones with the least inflated claims are given by $x_P(x) = x + 1/(2\kappa)$ and $x_D(x) = x - 1/(2\kappa)$. The amount of boasting is the same as asymptotically under the adversarial procedure. Under the inquisitorial procedure there is no finite starting point upon which the party can build in order to make his claims. For instance, under the adversarial procedure the plaintiff knows that if he testifies, the arbiter will expect the true x to be at least $\mu + \gamma$; in particular, the plaintiff will be believed to speak the truth if he claims $x_P = \mu + \gamma$. This opportunity is not available under the inquisitorial procedure: if the plaintiff wants to convince the arbiter that $x = \mu + \gamma$, he has to boost.

However, the inquisitorial arbiter has an additional option: he may require joint testimony. The idea is that, by forcing confrontation, both the plaintiff and the defendant will be induced to falsify less because boosting their claim is less productive. Under simultaneous testimony, the parties' equilibrium strategies $x_D(x)$ and $x_P(x)$ are again one-to-one functions span-

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ning the whole space. Hence, they can be inverted. When the arbiter observes the pair (x_D, x_P), he knows that the true x is equal to $x_D^{-1}(x_D) = x_P^{-1}(x_P)$ when the equality holds. When it does not hold, it must be that at least one party deviated from his equilibrium strategy. By the minimality condition, he believes that at most one did. In the least-cost separating profile, given the symmetry of lying costs, he assigns an equal chance to a deviation by the plaintiff or the defendant. He therefore adjudicates

$$\hat{x}(x_D, x_P) = \frac{1}{2}x_D^{-1}(x_D) + \frac{1}{2}x_P^{-1}(x_P).$$

The plaintiff chooses x_P to maximize

$$\frac{1}{2}x_D^{-1}(x_D) + \frac{1}{2}x_P^{-1}(x_P) - c_P(x_P, x).$$

The first-order conditions to this problem yields the differential equation

$$(x_P(x) - x)) \, x_P'(x) = \frac{1}{4\kappa}.$$

Similarly, the defendant's optimization problem yields

$$(x - x_D(x))) \, x_D'(x) = \frac{1}{4\kappa}.$$

The solutions with the smallest falsification are $x_P(x) = x + 1/(4\kappa)$ and $x_D(x) = x - 1/(4\kappa)$.

Under joint testimony, a party inflates his claim only half as much as he would if he were the only one to testify. The reason is that the arbiter now attaches to his testimony only half as much importance as he would under single testimony. A party falsifies less because lying is costly and it now has less influence on the arbiter's decision. Total lying costs are thus smaller under joint than under single testimony. Therefore, when γ is not too large, joint testimony is cheaper than single testimony even though fixed costs are duplicated.

3.3 Welfare comparison

Let us now compare the two procedures from a welfare point of view. This task is more tedious than under pure disclosure because signalling costs blur the picture. To start, note that average falsification costs under the adversarial procedure increase with the prior uncertainty about x: under high uncertainty large and costly signals get more weight than under low uncertainty.

Consider first low prior uncertainty and thus low average falsification costs. When the arbiter cares little about accuracy (θ low), he prefers the inquisitorial procedure and hears no testimony; when he cares a lot about accuracy (θ high), he also prefers the inquisitorial procedure and enforces testimony. For intermediate values of θ the adversarial procedure does a better job than the inquisitorial one. Therefore, for low prior uncertainty the results under signalling are similar to our results under pure disclosure.

Matters become more interesting when prior uncertainty and thus average falsification costs under the adversarial procedure are high. Recall that, unlike under pure disclosure, the inquisitorial arbiter may require joint testimony in the signalling set-up. He does so when the fixed cost γ is sufficiently small: joint testimony leads to lower signalling costs than single testimony which outweigh the duplicated fixed costs. It turns out that joint testimony is better than adversarial testimony for all values of θ if γ is below a threshold. Only for sufficiently high fixed costs and intermediate values of θ does the adversarial procedure do a better job than the inquisitorial one. For high prior uncertainty and low fixed cost the inquisitorial procedure is, therefore, superior to the adversarial procedure. [20]

To summarize our welfare results: In
- the pure disclosure framework and
- the falsification set-up
  — with either small prior uncertainty

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20. Viewed differently, for high prior uncertainty the inquisitorial procedure is more attractive under signalling than under pure disclosure.

— or with large prior uncertainty and high γ

the inquisitorial procedure is better for θ low or high. For intermediate values of θ the adversarial procedure does a better job.

In

- the falsification framework with high prior uncertainty and low γ

the inquisitorial procedure does better than the adversarial one for all values of θ.

We have ruled out any out-of-court settlement negotiations, as is typical in this literature. Yet, our agents have symmetric information—the framework where we should expect settlement.[21] Therefore, let us briefly discuss the possibility of settlement. Add a prior stage to our game where parties can negotiate a settlement payment S that the defendant pays to the plaintiff. If they settle, the game is over; if they don't, they enter the litigation subgame that we have analyzed in this paper. Let bargaining be costless and let the parties split the surplus evenly. The equilibria we have derived for the litigation subgame determine the parties' outside options (threat points) in the settlement stage.

Consider first the adversarial procedure in the falsification framework. Denote the equilibrium submission costs if the plaintiff reports by C_P^A and by C_D^A if the defendant reports. Then

$$
S = \begin{cases}
x - C_D^A/2, & \text{if } x \leq \mu + \gamma; \\
\mu, & \text{if } x \in (\mu - \gamma, \mu + \gamma); \\
x + C_P^A/2, & \text{if } x \geq \mu + \gamma.
\end{cases}
$$

Now consider the inquisitorial procedure when only the plaintiff reports. Then $x_P = x + 1/(2\kappa)$, $C_P^I = 1/(4\kappa^2)$, and $S = x + 1/(8\kappa^2)$. If both parties report, $x_P = x + 1/(4\kappa)$, $x_D = x - 1/(4\kappa)$, $C_P^I = C_D^I = 1/(16\kappa^2)$, and $S = x$.

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21. There are, however, situations where private settlement need not occur. Moreover, settlements, though privately beneficial, may be socially undesirable: the settlement amount may be too low/too high to provide proper ex ante incentives. For incentive reasons Judge Posner decided, e.g., in *Goesel v. Boley Intern.*, 738 F.3d 831 (2013), that the private settlement has to be made public. See, e.g., Hay and Spier (1998) for a further discussion of these points.

Under either procedure the parties settle so that there are no submission costs. Welfare only depends on error costs which are minimized by joint testimony under the inquisitorial procedure. Obviously, the assumption of equal bargaining power drives this result of implementing the first-best. Nevertheless, the example shows how pretrial bargaining can be incorporated in our set-up.

4   Concluding Remarks

We have derived the equilibrium testifying behavior under adversarial arbitration. When the true value of the amount at issue differs only slightly from the prior mean, the parties remain silent and do not spend resources on falsification. This comes at the price of incorrect decisions, but the social loss from inaccurate adjudication will also be small. Only when the true value differs sufficiently from the prior mean do parties testify. This enables correct decisions, yet at the expense of falsification.

Moreover, we have compared the adversarial with the inquisitorial procedure, taking into account submission costs and accuracy in adjudication. When wasteful influence expenditures are not too large, the inquisitorial procedure performs better when the arbiter has strong views about error costs; otherwise, the adversarial procedure does better. However, when falsification expenditures are an important component of the cost of testimony, by requiring joint testimony the inquisitorial procedure does better irrespective of the weight attached to accuracy in adjudication. [22]

We looked at extreme forms of both the adversarial and inquisitorial procedures. Under the former, our arbiter is passive and can just wait for testimony by the parties. Under the latter, the arbiter does not have the option to let the parties freely decide whether they want to testify: he can only either summon them to testify or refuse to hear them. Obviously, an

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22. Recall that most of our results rely on the normally distributed damages and the quadratic cost functions.

active arbiter who also has the option to let the parties freely testify would yield the best of both worlds.[23] On matters where accuracy has negligible social value, he would refuse to hear the parties. When accuracy has very large social value, he could summon one or both parties to testify. In intermediate cases, he could let the parties decide whether or not they want to testify. He then relies on the parties' superior private information about the true state to reach the best compromise between submission costs and accuracy. This is not unlike the justification often given for "managerial judges" who participate in activities such as pretrial discovery and settlement bargaining (see Schrag, 1999).

The arbiter in our framework adjudicates a single value that one party loses and the other party gains. Without this balanced-budget constraint, it is straightforward to elicit the truth without falsification. Suppose, for instance, the inquisitorial arbiter requires joint testimony. If both parties report the same number, the arbiter adjudicates this value. If the parties' reports differ, the judge punishes both of them. The judge could, e.g., punish them for perjury. However, perjury law is crude and ineffective; there is plenty of evidence indicating that false testimony is widespread in courts.[24] Furthermore, in the non-legal applications we mention in the introduction, the arbiter typically does not have the possibility to punish the parties.

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23. In Emons and Fluet (2018) the arbiter can bar, say, the defendant. The plaintiff is free to testify; the arbiter cannot, however, force the plaintiff to report.

24. See, e.g., Cooter and Emons (2004) and the references quoted therein.

Appendix 1

Welfare Comparison under Pure Disclosure

Comparing (1) and (2), it is obvious that the inquisitorial procedure does better when θ is either sufficiently small or sufficiently large. We have the following result.

Proposition 3: *For all $\gamma > 0$, there exists $0 < \underline{\theta}(\gamma) < \overline{\theta}(\gamma)$ such that $L^A < L^I$ if $\theta \in (\underline{\theta}(\gamma), \overline{\theta}(\gamma))$ and $L^I \leq L^A$ otherwise.*

Figure 2 depicts the social loss under each procedure as a function of θ. Under the inquisitorial procedure, the loss is the ex ante value of information, $\theta\sigma^2$, so long as this is smaller than the disclosure cost. Under the adversarial procedure, the loss is a straight line with slope less than σ^2 and vertical intercept less than the disclosure cost. In Appendix 2 we show that the L^A and L^I lines always intersect. Thus, for any positive disclosure cost, which procedure is better depends on the importance given to accuracy in adjudication. When the arbiter does not care too much about error costs or conversely when he cares a lot, he does better with the inquisitorial procedure where he fully controls which kind of costs he incurs. When the value of accuracy is in some intermediate range, the adversarial procedure does better.
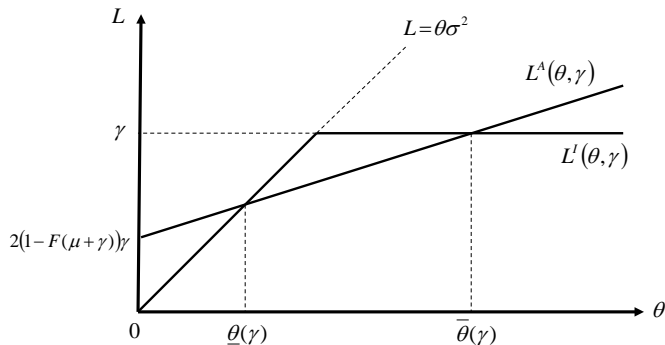

Figure 2: Inquisitorial versus adversarial

Our next result provides a characterization in (θ, γ)-space. To provide intuition, we first compare the adversarial procedure with a first-best scenario. Suppose a social planner observes x together with the parties. The planner cannot adjudicate,

which is the arbiter's responsibility, but he can force or forbid disclosure. When there is disclosure, the arbiter adjudicates $\hat{x} = x$; when there is no disclosure, he will rationally adjudicate $\hat{x} = \mu$. Obviously, the planner forces at most one party to disclose. He does so when the informational value is worth the cost, i.e., when $\theta(x - \mu)^2 \geq \gamma$. He will thus forbid disclosure when $x \in (\mu - \sqrt{\gamma/\theta}, \mu + \sqrt{\gamma/\theta})$ and he will enforce disclosure otherwise.

Therefore, when $\gamma = \sqrt{\gamma/\theta}$ or equivalently $\gamma = 1/\theta$, the parties' decisions under the adversarial procedure are socially efficient. When $\gamma > 1/\theta$, the plaintiff and defendant inefficiently remain silent for some values of x. When $\gamma < 1/\theta$, there is inefficient disclosure for some values. [25] Generically the amount of testimony is inefficient but is sometimes very close to the first best. Under the inquisitorial procedure, there is also either too much or too little testifying compared to the first best, but the outcome is then all or nothing.

Corollary 1: *If* $\theta \leq \underline{\theta}(\gamma)$, *then* $\gamma \in (\theta\sigma^2, 1/\theta)$; *if* $\theta \geq \overline{\theta}(\gamma)$, *then* $\gamma \in (1/\theta, \theta\sigma^2)$.

The result is illustrated in Figure 3. The lines $\gamma = 1/\theta$ and $\gamma = \theta\sigma^2$ partition the (θ, γ)-space into four regions. Together with Proposition 1, the Corollary tells us that the inquisitorial procedure does better than the adversarial one only in the interior of regions 2 and 4. Along the line $\gamma = 1/\theta$ the adversarial outcome yields the first-best. In region 1 there is too little disclosure under the adversarial procedure. However, there is no disclosure at all under the inquisitorial one, so that adversarial does better than inquisitorial. In region 3 we have too much disclosure under the adversarial procedure; yet there is even more disclosure under the inquisitorial one so that again adversarial does better than inquisitorial.

In region 2 there is too little disclosure under the adversarial procedure while disclosure always occurs under the inquisitorial procedure. From Proposition 3 we know that there exists an area such as I_2 where the inquisitorial procedure does better. In region 4 there is too much disclosure under the adversarial procedure and no disclosure at all under the inquisitorial one. From Proposition 3 again, there is an area such as I_4 where the inquisitorial procedure yields a smaller social cost. In Appendix 2 we show that the areas are as represented in Figure 3. In particular, the boundary of region I_2 gets asymptotically close to $\gamma = 1/\theta$ or $\gamma = \theta\sigma^2$ when γ becomes arbitrarily small or large.

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25. Inefficient non-disclosure arises for x in $(\mu + \sqrt{\gamma/\theta}, \mu + \gamma)$ or $(\mu - \gamma, \mu - \sqrt{\gamma/\theta})$; inefficient disclosure for x in $(\mu + \gamma, \mu + \sqrt{\gamma/\theta})$ or $(\mu - \sqrt{\gamma/\theta}, \mu - \gamma)$.
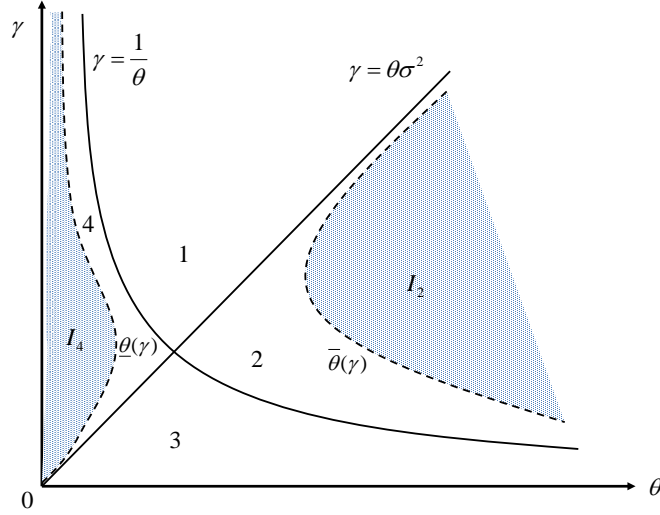
Figure 3: Optimal procedures under pure disclosure

Welfare Comparison under Falsification

The cost of testimony

We first collect some results that will be useful in our welfare comparison. Let us start with the inquisitorial procedure. Under single testimony, the lying cost of the testifying party is $k := \kappa \left(1/2\kappa\right)^2 = 1/4\kappa$. The total cost of testimony is $\gamma + k$. The easier it is to falsify, the larger the value of k. Under joint testimony, each party bears the cost $\gamma + \kappa \left(1/4\kappa\right)^2 = \gamma + k/4$. Summing over both parties yields the total cost of testimony $2\gamma + k/2$. Joint testimony is cheaper than single testimony when $2\gamma + k/2 < \gamma + k$ or equivalently $\gamma < k/2$. Thus, for any fixed cost, joint testimony will be cheaper than single testimony if falsification is sufficiently easy.

Consider next the adversarial procedure. Because we will be looking at the family of equilibria generated by different values of γ, we write the parties' strategies explicitly as a function of that parameter. For the plaintiff, the falsification cost at equilibrium is $v^P(x, \gamma) := \kappa \left(x_P(x, \gamma) - x\right)^2$, $x \geq \mu + \gamma$. From the previous discussion, we know that the function is increasing and concave in x with $v^P(\mu + \gamma, \gamma) = 0$ and $v^P(\infty, \gamma) = k$. For the defendant $v^D(x, \gamma) := \kappa \left(x - x_D(x, \gamma)\right)^2$, $x \leq \mu - \gamma$, where $v^D(\mu - \gamma, \gamma) = 0$ and $v^D(-\infty, \gamma) = k$. In Appendix 2 Corollary 2 summarizes some properties of the falsification expenditure functions.

Testimonies depend on γ only through the initial condition, i.e., the curves $x_D(x, \cdot)$ and $x_P(x, \cdot)$ simply shift horizontally when the fixed cost changes. When the parties are more reluctant to testify they also falsify less, everything else equal.

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Over the range where a party testifies, the average falsification expenditure is

$$\overline{v}(\gamma) := \int_{-\infty}^{\mu-\gamma} v^D(x,\gamma)\frac{f(x)}{F(\mu-\gamma)}\,dx = \int_{\mu+\gamma}^{\infty} v^P(x,\gamma)\frac{f(x)}{1-F(\mu+\gamma)}\,dx.$$

The equality follows from symmetry. The expected falsification expenditure under the adversarial procedure is $[F(\mu-\gamma)+1-F(\mu+\gamma)]\overline{v}(\gamma)$. It follows immediately from Corollary 2 that the expected falsification expenditure is decreasing in γ. Adding γ gives us the expected cost of testimony.

Finally, let us reconsider the plaintiff's incentive to file suit. Under the adversarial procedure, his expected payoff is $\mu - (1 - F(\mu + \gamma))(\gamma + \overline{v}(\gamma))$. Under the inquisitorial procedure, it is at worst $\mu - \gamma - k$. Under either procedure, a sufficient condition to sue is, therefore, $\mu > \gamma + k$.

Welfare comparison

To begin the analysis it is useful to take pure disclosure as a benchmark. Let the fixed cost of testimony under claim falsification be equal to the submission cost under pure disclosure. Several results follow immediately.

Consider the adversarial procedure. Even though the parties now attempt to boost their claims, their decisions whether or not to testify are the same as under pure disclosure: parties testify when $x \notin (\mu - \gamma, \mu + \gamma)$. Because the arbiter infers the truth from the parties' testimony, the social loss from inaccurate adjudication is equal to the loss under pure disclosure. However, testimony is more costly.

When parties may falsify, testimony under the inquisitorial procedure is also more costly than in the pure disclosure framework. Because the arbiter trades-off the cost of testimony against error costs, he will be more reluctant to allow testimony than under pure disclosure. Denote by $\overline{\gamma}(\theta)$ the frontier between testimony and no-testimony in the (θ, γ)-space, i.e., testimony is allowed only when $\gamma \leq \overline{\gamma}(\theta)$. With pure disclosure, $\overline{\gamma}(\theta) = \theta\sigma^2$. When claims are inflated, the cost of testimony is either $\gamma + k$ (single testimony) or $2\gamma + k/2$ (joint testimony), whichever is cheaper. Then $\overline{\gamma}(\theta) = \max\left[\theta\sigma^2 - k, \theta\sigma^2/2 - k/4, 0\right]$; see Figures 4 and 5. Under the inquisitorial procedure, adjudication will, therefore, more often be inaccurate when parties may falsify than when they may not.

Finally, the cost of testimony will differ between procedures when parties may boost. When single testimony is optimal under the inquisitorial procedure, the cost of testimony is larger than with the adversarial procedure. When joint testimony is best, it is not clear at first sight which procedure is cheaper.

The social loss under the inquisitorial procedure is

$$L^I(\theta, \gamma) = \min\left\{\theta\sigma^2, \gamma + k, 2\gamma + k/2\right\}. \tag{8}$$

Under the adversarial procedure, it is

$$L^A(\theta, \gamma) = \theta \int_{\mu-\gamma}^{\mu+\gamma} (x-\mu)^2 f(x)\, dx + [F(\mu-\gamma) + 1 - F(\mu+\gamma)](\gamma + \overline{v}(\gamma)). \quad (9)$$

Taking γ as given, (8) and (9) describe functions of θ similar to the ones depicted in Figure 2. Indeed, when single testimony is optimal under the inquisitorial procedure, $L^I(\theta, \gamma) = \min\left\{\theta\sigma^2, \gamma + k\right\}$. Then the L^I and L^A lines will be as drawn in Figure 2. Specifically, because $\overline{v}(\gamma) < k$, an argument similar to the one used in Proposition 1 shows that the lines necessarily intersect. Thus, the inquisitorial procedure will do better for either small or large values of θ, while the adversarial procedure will do better for values in between. The same argument cannot be used, however, when joint testimony is best, i.e., when $L^I(\theta, \gamma) = \min\left\{\theta\sigma^2, 2\gamma + k/2\right\}$. As we will show below, it is then possible that the L^I and L^A lines do not intersect. When this occurs, the inquisitorial procedure does better for all values of θ.

In Figures 4 and 5, the lines $\gamma = 1/\theta$ and $\gamma = \overline{\gamma}(\theta)$ are used to partition the (θ, γ)-space into four regions, as was done in section 2. In Figure 4, the lines intersect on the no testimony-joint testimony frontier of the inquisitorial procedure. In Figure 5, they intersect on the no testimony-single testimony frontier. In the latter case, region 3 defined by $\gamma \leq \min\{1/\theta, \overline{\gamma}(\theta)\}$ has been further partitioned into two parts: the subregion 3a is for the case where $\gamma \geq k/2$ so that single testimony is required under the inquisitorial procedure; the subregion 3b is for the case where $\gamma < k/2$ and joint testimony is required. In the next result, regions 2 and 3 are taken to be closed sets, i.e., they include their frontier.

Lemma 2: $L^A < L^I$ in regions 1 and 3a. If $\overline{v}(0) \leq k/2$, $L^A < L^I$ in the whole of region 3.
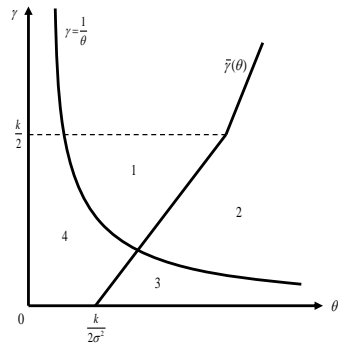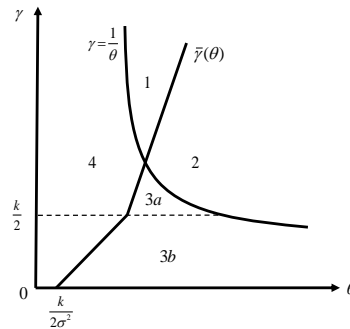

Figure 4: Large values of k Figure 5: Smaller values of k

The argument for region 1 is similar to the one used under pure disclosure, except that falsification costs must now be taken into account; in Appendix 2 we

show that when parties testify under the adversarial procedure, the social value of information always exceeds the cost of testimony. The argument for region 3a (when it is non-empty) is also similar to the one used for region 3 under pure disclosure: the adversarial procedure does better because unwarranted testimony arises less often than under the inquisitorial procedure, to which must now be added the fact that the cost of testimony in region 3a is smaller under the adversarial than under the inquisitorial procedure. In the rest of region 3, that condition cannot be guaranteed. It does hold, however, when $\overline{v}(0) \leq k/2$.

The foregoing condition plays an important role in what follows. Under the adversarial procedure the expected falsification expenditure is

$$[F(\mu - \gamma) + 1 - F(\mu + \gamma)]\overline{v}(\gamma)$$

which is decreasing in γ. In the limiting case where γ tends to zero, the parties always testify under the adversarial procedure and the expression reduces to $\overline{v}(0)$. $\overline{v}(0)$ is thus the upper bound of the expected falsification expenditure under adversarial testimony. When $\overline{v}(0) \leq k/2$, the expected falsification expenditure under the adversarial procedure is therefore always smaller than under the inquisitorial one, for any value of γ. The same holds for the expected total cost of testimony including the fixed cost. When the inequality is reversed, however, there will be a range of γ-values where the inquisitorial procedure is cheaper because it yields smaller falsification costs.

From the preceding section we know that over the range where a party testifies, a party's falsification cost under the adversarial procedure varies between zero and k. When prior beliefs are very precise (i.e., when σ^2 is very small), most of the probability weight will be concentrated close to the mean of the distribution. Because $v^D(\mu, 0) = v^P(\mu, 0) = 0$, we will have $\overline{v}(0) < k/2$. Conversely, when prior beliefs are very diffuse, most of the probability weight will be on values of x far from the mean and we will have $\overline{v}(0) > k/2$.

Let us now turn to our last result that shows in (θ, γ)-space which procedure is optimal.

Proposition 4:

   i) If $\overline{v}(0) \leq k/2$, then for all $\gamma > 0$ there exists $\overline{\theta}(\gamma) > \underline{\theta}(\gamma) > 0$ such that $L^A < L^I$ if $\theta \in (\underline{\theta}(\gamma), \overline{\theta}(\gamma))$ and $L^I \leq L^A$ otherwise.

   ii) If $\overline{v}(0) > k/2$, then there exists $\hat{\gamma} > 0$ such that $L^I \leq L^A$ for all θ if $\gamma \leq \hat{\gamma}$; if $\gamma > \hat{\gamma}$ there exists $\underline{\theta}(\gamma)$ and $\overline{\theta}(\gamma)$ as in i).

The first part of the Proposition is illustrated in Figure 6. The result is similar to the one obtained under pure disclosure. However, the area I_2 is smaller whereas I_4 is larger than the corresponding areas under pure disclosure.
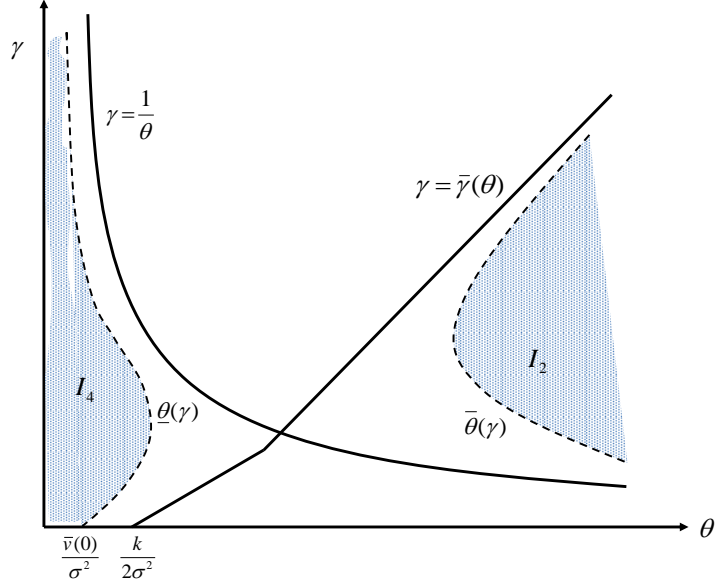
Figure 6: Small prior uncertainty

In region 2 the inquisitorial arbiter requires testimony. This region is smaller than under pure disclosure; the inquisitorial arbiter is more reluctant to require testimony because testimony is now more costly because of falsification costs. This is captured by the lower upper bound $\overline{\gamma}(\theta)$ for region 2 compared to pure disclosure. In addition, when $\overline{v}(0) \leq k/2$, the cost of testimony under the adversarial procedure is smaller than under the inquisitorial procedure. Hence, the area within region 2 where the inquisitorial does better is smaller: for a given θ, the inquisitorial procedure will do better only within a smaller range of γ-values. [26] In region 4 the inquisitorial arbiter refuses to hear any testimony at all, yielding the social loss $L^I = \theta\sigma^2$. The area I_4 is larger than under pure disclosure because the cost of adversarial testimony is now larger. For a given θ, the inquisitorial procedure will now do better within a larger range of γ-values.

The second part of Proposition 4 is illustrated in Figure 7. This differs markedly from the pure disclosure set-up. The inquisitorial procedure does better in the shaded area. To see how this area is obtained, suppose that the inquisitorial arbiter does not have the option of requiring joint testimony, so that $L^I(\theta, \gamma) = \min\left\{\theta\sigma^2, \gamma + k\right\}$ irrespective of the values of γ and k. Then the inquisitorial procedure does better in the shaded area below the two dashed curves,

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26. One can show that I_2 is bounded below by the curve $\gamma = g(\theta) := (1+\sqrt{1+\theta/k})/2\theta$. This curve is above the $\gamma = 1/\theta$ curve but tends to it asymptotically when θ becomes arbitrarily large.

i.e., either for small for or large values of θ as in part i) of Proposition 4.
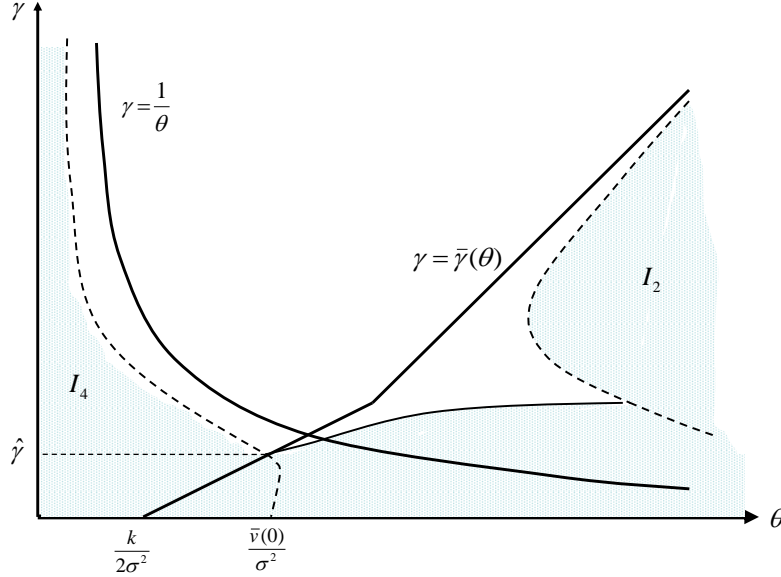

Figure 7: Large prior uncertainty

When the option of joint testimony becomes available, the rest of the shaded area must be added: confronting the parties is cheaper than single testimony for $\gamma < k/2$. Therefore, for all $\gamma \leq \hat{\gamma}$, the inquisitorial procedure does better than the adversarial one for all values of θ; here $\hat{\gamma}$ is the value of γ at which the shaded area crosses the joint testimony-no testimony frontier. In Appendix 2 we show that the relevant areas are as depicted.

Appendix 2

Proof of Proposition 1. Given the judge's concern about correct decisions, he adjudicates $\hat{x} = x$ when he hears testimony. When he hears no testimony suppose he adjudicates some $\hat{x} = \nu$. Given the judge's behavior, the parties will never testify together. If, say, the defendant deviates to no testimony while the plaintiff testifies, the defendant doesn't change the arbiter's decision and saves the submission cost γ.

Suppose the defendant is silent. The plaintiff will disclose if $x - \gamma \geq \nu$; otherwise, he is better off remaining silent. Likewise, the defendant will disclose if $-x - \gamma \geq -\nu$ and is silent otherwise given the plaintiff is silent. Therefore, the arbiter knows

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that $x \in (\nu - \gamma, \nu + \gamma)$ when he hears no testimony. To minimize error he adjudicates the posterior mean. Given that $f(x)$ is normal, $\nu = E(x | \nu - \gamma < x < \nu + \gamma)$ is possible only if $\nu = \mu$. ∎

Proof of Lemma 1.

(i) Suppose first that both parties always testify. Let (x'_P, x'_D) be the equilibrium pair at x'. Because x' is revealed, it follows from monotonicity that either $x_P(x)$ or $x_D(x)$ is strictly increasing at x'. Let this be true of $x_P(x)$. If D deviates to $\emptyset_D$, by minimality the arbiter then infers x' from the observation of $(x'_P, \emptyset_D)$. Therefore, D is better off because he saves on the cost of testimony without affecting the arbiter's decision.

Next, suppose P always testifies but D does so only at $x \leq x_D^0$. If $x_D(x)$ is strictly increasing at some $x' < x_D^0$, the preceding argument shows that P would be better off deviating to $\emptyset_P$. So it must be that $x_D(x)$ is constant for all $x \leq x_D^0$, the true state being revealed only through $x_P(x)$. If P deviates to $\emptyset_P$ at some $x \leq x_D^0$, by minimality the arbiter will adjudicate some $\nu \leq x_D^0$. But then P saves on the cost of testimony and is better off deviating whenever $x < \nu$. Finally, suppose P always testifies but D never does. The argument is then similar: if P deviates to $\emptyset_P$ at some x, the arbiter will adjudicate some ν and P will be better off if x is sufficiently small.

We conclude that D testifies for $x \leq x_D^0$ and is otherwise silent, P testifies for $x \geq x_P^0$ and is otherwise silent, where x_D^0 and x_P^0 are finite.

(ii) It remains to show that $x_D^0 < x_P^0$. Suppose on the contrary that there is some range $[x_P^0, x_D^0]$ where both parties testify. Then the same argument as in the first paragraph of part (i) above shows that this yields a contradiction. ∎

Proof of Proposition 2. We briefly complete the argument in the text. Rather than attempting to solve (4) and (5) directly, it is easier to work with the equations expressed in terms of the inverse of $x_P(x)$ and $x_D(x)$, which we write $x(x_P)$ and $x(x_D)$ respectively. The differential equations then become:

$$2\kappa(x_P - x(x_P)) = x'(x_P), \tag{10}$$

$$2\kappa(x(x_D) - x_D) = x'(x_D). \tag{11}$$

The general solution to (10), given the condition $x(x_P) \leq x_P$, is

$$x = x_P - \frac{1 - Ke^{-2\kappa x_P}}{2\kappa}$$

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for some constant K. Choosing the constant to satisfy the initial condition $x(x_P^0) = x_P^0$ yields

$$x = x_P - \frac{1 - e^{-2\kappa(x_P - x_P^0)}}{2\kappa}.$$

Similarly,

$$x = x_D + \frac{1 - e^{-2\kappa(x_D^0 - x_D)}}{2\kappa}.$$

It remains to determine the constants x_P^0 and x_D^0. The argument is the same as in Proposition 1. When neither party submits, the arbiter's posterior mean is

$$\varphi_\emptyset := E\left(x \mid \emptyset_P, \emptyset_D\right) = E\left(x \mid x_D^0 < x < x_P^0\right). \tag{12}$$

At $x = x_P^0$ party P is just indifferent between submitting and not submitting. If the party submits, the true state is revealed at the cost of γ. If the party does not submit, the arbiter adjudicates $\varphi_\emptyset$. Thus party P is indifferent if $x_P^0 - \gamma = \varphi_\emptyset$. Using the same argument, when $x = x_D^0$, party D is indifferent between submitting and not if $-x_D^0 - \gamma = -\varphi_\emptyset$. Combining with (12) yields

$$\varphi_\emptyset = E\left(x \mid \varphi_\emptyset - \gamma < x < \varphi_\emptyset + \gamma\right).$$

Thus, the updated expectation given that x is in the interval $[\varphi_\emptyset - \gamma, \varphi_\emptyset + \gamma]$ must be the mid point $\varphi_\emptyset$. Given the normal density, this is possible only if $\varphi_\emptyset$ equals the prior mean μ. Consequently, $x_P^0 = \mu + \gamma$, $x_D^0 = \mu - \gamma$.

Finally, we consider the arbiter's beliefs for out-of-equilibrium moves. We discuss only the beliefs that prevent a unilateral deviation by P; deviations by D would be dealt with in the same way. There are two relevant situations:

(i) The true state is $x > \mu - \gamma$; the equilibrium pair is $(\emptyset_P, \emptyset_D)$ if $x < \mu + \gamma$, it is $(x_P(x), \emptyset_D)$ with $x_P(x) \geq \mu + \gamma$ if $x \geq \mu + \gamma$. Suppose the arbiter observes $(x_P, \emptyset_D)$ with $x_P < \mu + \gamma$. By minimality, he believes that P has deviated while D played his equilibrium strategies. Hence he puts unit probability on the event $x > \mu - \gamma$. One possibility is that he infers $x = \mu$, in which case the deviation by P is clearly unprofitable.

(ii) The true state is $x \leq \mu - \gamma$ so that the equilibrium pair is $(\emptyset_P, x_D(x))$, where $x_D(x) \leq \mu - \gamma$. Suppose the arbiter observes $(x_P, x_D(x))$ with $x_P < \mu + \gamma$. By minimality, he believes that P has deviated while D played at equilibrium. Hence he disregards x_P and infers the state from the play of x_D alone, which clearly makes the deviation unprofitable. Next, suppose the arbiter observes $(x_P, x_D(x))$ with $x_P \geq \mu + \gamma$. Then the arbiter does not know who has deviated. By minimality, he believes that at most one did. One possibility is that he assigns an equal chance to either possibility and therefore adjudicates $x_D^{-1}(x_D)/2 + x_P^{-1}(x_P)/2$. P then earns the payoff

$$\pi := \tfrac{1}{2}x + \tfrac{1}{2}x_P^{-1}(x_P) - \gamma - \kappa(x_P - x)^2,$$

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which must be compared with his equilibrium payoff of x. Because $x_P(\cdot)$ satisfies (4),

$$\frac{\partial \pi}{\partial x_P} = \frac{1}{2}\frac{\partial x_P^{-1}(x_P)}{\partial x_P} - 2\kappa(x_P - x) = -\kappa(x_P - x) < 0.$$

Hence, we need only check whether a deviation to $x_P = \mu + \gamma$ might be profitable. The payoff is then

$$\pi = \tfrac{1}{2}x + \tfrac{1}{2}(\mu + \gamma) - \gamma - \kappa(\mu + \gamma - x)^2.$$

This is increasing in x, where by assumption $x \leq \mu - \gamma$. If a deviation to $x_P = \mu + \gamma$ is ever profitable, it must therefore be profitable when $x = \mu - \gamma$. The payoff is then $\pi = \mu - \gamma - \kappa(2\gamma)^2 < \mu - \gamma = x$. It follows that such a deviation can never be profitable. ■

Proof of Proposition 3. We complete the argument in the text by showing that $L^A(\gamma/\sigma^2, \gamma) < L^I(\gamma/\sigma^2, \gamma)$, i.e.,

$$(\gamma/\sigma^2) \int_{\mu-\gamma}^{\mu+\gamma} (x - \mu)^2 f(x)\,dx + \gamma[F(\mu - \gamma) + 1 - F(\mu + \gamma)] < \gamma$$

or equivalently

$$\int_{\mu-\gamma}^{\mu+\gamma} (x - \mu)^2 \left(\frac{f(x)}{F(\mu + \gamma) - F(\mu - \gamma)}\right) dx < \int_{-\infty}^{\infty} (x - \mu)^2 f(x)\,dx.$$

Given symmetry, the distribution $f(x)$ is a mean-preserving spread of the conditional distribution $f(x)/\left(F(\mu + \gamma) - F(\mu - \gamma)\right)$. The inequality follows from the strict convexity of $(x - \mu)^2$ with respect to x. ■

Proof of Corollary 1. If $\theta \leq \underline{\theta}(\gamma)$, it follows directly from Proposition 3 and Figure 2 that $\gamma > L^A(\theta, \gamma) \geq L^I(\theta, \gamma) = \theta\sigma^2$. Similarly, if $\theta \geq \overline{\theta}(\gamma)$, it follows that $\theta\sigma^2 > L^A(\theta, \gamma) \geq L^I(\theta, \gamma) = \gamma$. Using symmetry, the inequality $L^A(\theta, \gamma) \geq \theta\sigma^2$ is easily seen to be equivalent to

$$\int_{\mu+\gamma}^{\infty} [\gamma - \theta(x - \mu)^2] f(x)\,dx \geq 0. \tag{13}$$

The inequality $L^A(\theta, \gamma) \geq \gamma$ is equivalent to

$$\int_{\mu}^{\mu+\gamma} [\gamma - \theta(x - \mu)^2] f(x)\,dx \leq 0. \tag{14}$$

Now, observe that

$$\int_{\mu+\gamma}^{\infty} [\gamma - \theta(x - \mu)^2] f(x)\,dx < \int_{\mu+\gamma}^{\infty} \gamma(1 - \gamma\theta) f(x)\,dx \tag{15}$$

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and

$$\int_{\mu}^{\mu+\gamma} [\gamma - \theta(x-\mu)^2]f(x)\,dx > \int_{\mu}^{\mu+\gamma} \gamma(1-\gamma\theta)f(x)\,dx. \tag{16}$$

If $\gamma\theta \geq 1$, (15) implies that (13) cannot hold. Thus, $\theta \leq \underline{\theta}(\gamma)$ implies $\gamma < 1/\theta$ as well as $\gamma > \theta\sigma^2$. If $\gamma\theta \leq 1$, (16) implies that (14) cannot hold. Thus, $\theta \geq \overline{\theta}(\gamma)$ implies $\gamma > 1/\theta$ as well as $\gamma < \theta\sigma^2$. ■

Properties of the functions $\underline{\theta}(\gamma)$ and $\overline{\theta}(\gamma)$. $\underline{\theta}(\gamma)$ is the unique solution to $L^A(\theta,\gamma) = \theta\sigma^2$. Because $L^A(\theta,0) = 0$, $\underline{\theta}(0) = 0$. Moreover, for any $\gamma > 0$, there exists θ sufficiently small such that (γ,θ) is in I_4. Hence, the curve $\underline{\theta}(\gamma)$ is tangent to the curve $\gamma = \theta\sigma^2$ at $\gamma = \theta = 0$. Indeed,

$$\underline{\theta}'(\gamma) = \frac{L_\gamma^A(\gamma,\theta)}{\sigma^2 - 2\int_{\mu}^{\mu+\gamma}(x-\mu)^2 f(x)\,dx}$$

where

$$L_\gamma^A(\gamma,\theta) = 2\left[1 - F(\mu+\gamma)\right] - 2\gamma(1-\theta\gamma)f(\mu+\gamma).$$

Hence, $\underline{\theta}'(0) = 1/\sigma^2$. It is easily verified that $\underline{\theta}''(0) < 0$. Similarly, for γ large, the curve cannot intersect the vertical axis. Hence, it must extend upwards as shown.

$\overline{\theta}(\gamma)$ is the unique solution to $L^A(\gamma,\theta) = \gamma$. For any $\gamma > 0$, there exists θ sufficiently large such that (γ,θ) is in I_2. Therefore, the curve must extend indefinitely downwards bounded only by $\gamma = 1/\theta$, and indefinitely upwards bounded only by $\gamma = \theta\sigma^2$. Indeed

$$\overline{\theta}'(\gamma) = \frac{1 - L_\gamma^A(\gamma,\theta)}{2\int_{\mu}^{\mu+\gamma}(x-\mu)^2 f(x)\,dx},$$

When γ and θ tend to infinity, $\overline{\theta}'(\gamma)$ tends to $1/\sigma^2$.

Proof that pooling with no testimony is no equilibrium under the Grossman-Perry refinement.

Take some $x_P > \mu + \gamma$. Suppose P expects a judgement $\hat{x} \geq \gamma$. Then P makes no loss when the true x satisfies $\hat{x} - \gamma - \kappa(x_P - x)^2 \geq \mu$ and benefits if the inequality is strict. Let $\underline{x}$ and $\overline{x}$ be the solutions for the equality in this condition and write $\underline{x} = x_P - h(\hat{x})$, $\overline{x} = x_P + h(\hat{x})$, where $h(\hat{x}) = \sqrt{(\hat{x} - (\mu+\gamma))/\kappa}$. Note that $h(\hat{x})$ is strictly increasing with $h(\mu+\gamma) = 0$.

Now define $\phi(\hat{x}) = E(x | x_P - h(\hat{x}) \leq x \leq x_P + h(\hat{x}))$. Note that $\phi(\mu+\gamma) = x_P > \mu + \gamma$. For x_P sufficiently large, $x_P - h(\hat{x}) > \mu$ so that $f(x)$ is decreasing over the interval. Because x_P is the midpoint, it then follows that $\phi(\hat{x}) < x_P$. In particular $\phi(x_P) < x_P$. By continuity, jusing the mean value theorem, there exist $\hat{x}^* \in (\mu+\gamma, x_P)$ such that $\phi(\hat{x}^*) = \hat{x}^*$. Thus, when $x \in (x_P - h(\hat{x}^*), x_P + h(\hat{x}^*))$, P would do strictly better by reporting x_P expecting $\hat{x}^*$ and the judge would be right to infer the posterior mean $\hat{x}^*$. ■

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The next result lists some useful properties of the falsification costs at equilibrium.

Corollary 2. $v_x^P(x,\gamma) \in (0,1]$ with $v_x^P(\mu + \gamma, \gamma) = 1$; $v_x^D(x,\gamma) \in [-1,0)$ with $v_x^D(\mu - \gamma, \gamma) = -1$; $v_{xx}^i(x,\gamma) < 0$, $v_\gamma^i \in [-1,0)$, $i = D, P$, with $v_\gamma^D(\mu - \gamma, \gamma) = v_\gamma^P(\mu + \gamma, \gamma) = -1$.

Proof. Let $x_P(x,\gamma)$ be the solution to equation (6) with initial condition $x_P(\mu + \gamma, \gamma) = \mu + \gamma$. Define

$$\varphi(x,\gamma) := x_P(x,\gamma) - x, \quad x \geq \mu + \gamma, \ \gamma \geq 0,$$

where φ is the "claim inflation function". From (4) and given the initial condition, $\varphi(\mu + \gamma, \gamma) = 0$ and

$$\varphi(x,\gamma)(1 + \varphi_x(x,\gamma)) = \frac{1}{2\kappa}. \tag{21}$$

Then $\varphi(x,\gamma) < 1/(2\kappa)$ and $\varphi_{xx} < 0$ with $\varphi_x(\mu + \gamma, \gamma) = \infty$ and $\varphi_x(\infty, \gamma) = 0$.

The plaintiff's falsification expenditure is

$$\nu^P(x,\gamma) = \kappa \varphi(x,\gamma)^2, \quad x \geq \mu + \gamma.$$

For $x \geq \mu + \gamma$ and using (21),

$$\nu_x^P(x,\gamma) = 2\kappa \varphi(x,\gamma)\varphi_x(x,\gamma) = 1 - 2\kappa\varphi(x,\gamma)$$

Therefore $\nu_{xx}^P < 0$ and $\nu_x^P(x,\gamma) \in (0,1]$. From (6) it is easily verified that $\partial x_P / \partial\gamma = 1 - \partial x_P / \partial x$, implying $\varphi_\gamma = -\varphi_x$. Therefore

$$\nu_\gamma^P(x,\gamma) = -(1 - 2\kappa\varphi(x,\gamma))$$

with $\nu_\gamma^P(x,\gamma) \in (-1,0]$. At $x = \mu + \gamma$, the preceding partial derivative is not defined. We use

$$\nu_\gamma^P(\mu + \gamma, \gamma) = \lim_{x \downarrow \mu + \gamma} \nu_\gamma^P(x,\gamma) = -1.$$

For D, $x - x_D(x,\gamma) = \varphi(x,\gamma)$ and the argument is the same. ∎

Proof of Lemma 2.

(i) In Region 1, $\theta\gamma \geq 1$ and $L^I(\theta,\gamma) = \theta\sigma^2$. Using symmetry,

$$\Delta(\theta,\gamma) := L^I(\theta,\gamma) - L^A(\theta,\gamma) = 2 \int_{\mu+\gamma}^{\infty} \psi(x)f(x)\,dx,$$

where

$$\psi(x) := \theta(x - \mu)^2 - (\gamma + v^P(x,\gamma)), \ x \geq \mu + \gamma.$$

It suffices to show that $\psi(x)$ is always positive. Applying Corollary 2, $\psi''(x) > 0$. Moreover, $v^P(\mu + \gamma, \gamma) = 0$ and $v_x^P(\mu + \gamma, \gamma) = 1$. Hence

$$\psi(\mu + \gamma) = \gamma(\theta\gamma - 1) \geq 0,$$

$$\psi'(\mu + \gamma) = 2\theta\gamma - v_x^P(\mu + \gamma, \gamma) \geq 1.$$

Therefore $\psi(x) > 0$ for all $x > \mu + \gamma$.

(ii) In Region 3a, if it exists, $\theta\gamma \leq 1$ and $L^I(\theta, \gamma) = \gamma + k$. Then

$$
\begin{aligned}
\frac{\Delta(\theta, \gamma)}{2} &= \int_\mu^{\mu+\gamma} \left[\gamma + k - \theta(x-\mu)^2\right] f(x)\, dx + \int_{\mu+\gamma}^\infty \left(k - v^P(x, \gamma)\right) f(x)\, dx \\
&> \int_\mu^{\mu+\gamma} \left[k + \gamma(1 - \theta\gamma)\right] f(x)\, dx + \int_{\mu+\gamma}^\infty \left(k - v^P(x, \gamma)\right) f(x)\, dx > 0.
\end{aligned}
$$

(iii) Consider now the area defined by $\gamma \leq k/2$ and $\gamma \leq \min\{\overline{\gamma}(\theta), 1/\theta\}$. When $\overline{\gamma}(\theta)$ and $\gamma = 1/\theta$ intersect at some $\gamma \leq k/2$, this area is the whole of region 3; when the intersection occurs at some $\gamma > k/2$, the area is region 3b. In either case, $L^I(\theta, \gamma) = 2\gamma + k/2$. Hence

$$\frac{\Delta(\theta, \gamma)}{2} = \gamma + \frac{k}{4} - \int_\mu^{\mu+\gamma} \theta(x-\mu)^2 f(x)\, dx - \int_{\mu+\gamma}^\infty \left(\gamma + v^P(x, \gamma)\right) f(x)\, dx.$$

When $\overline{v}(0) \leq k/2$,

$$\Delta(\theta, 0) = \frac{k}{2} - \overline{v}(0) \geq 0.$$

Differentiating with respect to γ,

$$\frac{\Delta_\gamma(\theta, \gamma)}{2} = 1 + \gamma(1 - \theta\gamma)f(\mu + \gamma) - \int_{\mu+\gamma}^\infty (1 + \nu_\gamma^P(x, \gamma))f(x)\, dx$$

By Corollary 2, $\nu_\gamma^P(x, \gamma) < 0$ so that

$$
\begin{aligned}
\frac{\Delta_\gamma(\theta, \gamma)}{2} &> 1 + \gamma(1 - \theta\gamma)f(\mu + \gamma) - \int_{\mu+\gamma}^\infty f(x)\, dx \\
&= \gamma(1 - \theta\gamma)f(\mu + \gamma) + F(\mu + \gamma) > 0.
\end{aligned}
$$

Therefore $\Delta(\theta, \gamma) > 0$ for any positive γ. ∎

Proof of Proposition 4. Let $\Delta(\theta, \gamma) = L^I(\theta, \gamma) - L^A(\theta, \gamma)$. From the argument in the text, we know that: for any $\gamma > 0$, if $\Delta(\theta, \gamma) > 0$ for some θ', there exists $\overline{\theta}(\gamma) > \underline{\theta}(\gamma) > 0$ such that $\Delta(\theta, \gamma) > 0$ if $\theta \in (\underline{\theta}(\gamma), \overline{\theta}(\gamma))$ and $L^I \leq L^A$ otherwise.

i) By Lemma 2, when $\overline{v}(0) \leq k/2$, $\Delta(\theta, \gamma) > 0$ in the regions 1 and 3. For any γ, one can choose θ so that (θ, γ) is in region 1 or in region 3. Hence the

condition $\Delta > 0$ can always be satisfied for some θ, which proves the first part of the Proposition.

ii) Suppose now $\overline{v}(0) > k/2$. For any $\gamma \geq k/2$, one can always choose θ such that (θ, γ) is in region 1 (as in Figure 4) or is in either region 1 or region 3a (as in Figure 5). The same argument as in i) can then be applied. We therefore restrict attention to $\gamma < k/2$. We then have $L^I(\theta, \gamma) = \min\{(\theta\sigma^2, 2\gamma + k/2\}$. Moreover

$$\Delta(\theta, 0) = \min\{\theta\sigma^2, k/2\} - \overline{v}(0) < 0.$$

By continuity, therefore, $\Delta(\theta, \gamma) \leq 0$ for all θ if γ is not too large. We now characterize the area where the preceding inequality holds.

When $2\gamma + k/2 \leq \theta\sigma^2$ (i.e., when joint testimony is preferred to no testimony), $\Delta(\theta, \gamma) \leq 0$ implies $\Delta(\theta', \gamma) < 0$ for all $\theta' > \theta$. When $2\gamma + k/2 \geq \theta\sigma^2$ (i.e., when no testimony is preferred), $\Delta(\theta, \gamma) \leq 0$ implies $\Delta(\theta', \gamma) < 0$ for all $\theta' < \theta$. In particular, for points on the boundary between joint testimony and no testimony, $\Delta(\theta, \gamma) \leq 0$ implies $\Delta(\theta', \gamma) < 0$ for all $\theta' \neq \theta$.

On the boundary, the difference between the social losses is $\Delta(\theta, \overline{\gamma}(\theta))$, where $\overline{\gamma}(\theta) = \theta\sigma^2/2 - k/4$. At the horizontal intercept, $\Delta = k/2 - \overline{v}(0) < 0$. Let θ_0 be the smallest value at which $\Delta(\theta, \overline{\gamma}(\theta))$ changes sign, from negative to positive. Such a θ_0 necessarily exists because $\Delta > 0$ when $\overline{\gamma}(\theta)$ crosse the $\gamma = 1/\theta$ curve as in Figure 4 or the $\gamma = k/2$ line as in Figure 5. The critical $\hat{\gamma}$ referred to in the Proposition equals $\overline{\gamma}(\theta_0)$, i.e., $\Delta(\theta, \gamma) \leq 0$ for all θ can be true only for $\gamma \leq \overline{\gamma}(\theta_0)$.
∎

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