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Good Times, Bad Times, and Vertical Upstream Integration

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Abstract

We consider a set of downstream firms each of which has a stochastic requirement for a particular input. Downstream firms can produce the input themselves yet do not trade it. Upstream firms produce the input to sell it through a Walrasian market to downstream firms. Efficient pooling of capacities requires the input to be produced by upstream firms and traded in the market. Yet, downstream firms will always vertically integrate. By producing some of its own input needs, a downstream firm cuts down on aggregate input demand thus depressing prices in the market.

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I. Introduction

The traditional business explanation for vertical upstream integration is that firms want to assure their supply of inputs (Porter (1980)). For example, Chandler (1969, p. 37) argues in his discussion of the history of the largest U.S. companies that the strategy for vertical integration had come from the desire to have a more certain supply of stocks, raw materials, and other supplies. In a similar spirit, the transactions cost literature argues that uncertainty can make it difficult to deal in factor markets thus creating an incentive for vertical upstream integration in order to bypass these problems by transferring goods internally (see, e.g., Coase (1937), Malmgren (1961), and Williamson (1971)).

Despite this extensive informal literature, formal analyses on the effects of uncertainty about input supplies as an incentive for vertical upstream integration are rather rare. Arrow (1975) analyzes a model where vertically integrated firms obtain information on the input's supply conditions earlier than non-integrated firms. This information advantage creates a tendency toward complete vertical upstream integration.

Green (1986) considers a model where downstream firms face no uncertainty in their product market and sell all of their output at the exogenous market price. The input market is beset by exogenous stochastic demand. Input prices are fixed so that downstream firms may be rationed. To avoid rationing and to internalize the price system, downstream firms tend to fully integrate even though they are (slightly) less efficient than upstream firms. By additionally taking traders' risk attitudes into account, Hendrikse and Peters (1989) obtain partial vertical integration as an equilibrium market structure in a setup along the lines of Green.

Carlton (1979) analyzes a model where uncertainty from the product market transmits to the input market. Again, fixed prices prevail on the input market so that rationing may occur. To rule out full integration, Carlton assumes that an integrated firm cannot sell its input on the market and may, therefore, be stuck with input for which it has no use. The equilibrium market structure is characterized by partial vertical upstream integration. Risk averse downstream firms wish to secure their high probability demand.

Closest to our analysis is a paper by Bolton and Whinston (1993). They consider a setup where a single upstream unit produces an input used by two downstream firms. The upstream firm has random capacity so that supplies may be insufficient to meet both downstream firms' needs. As in Grossman and Hart (1986), ex ante contracts can only be written about the allocation of ownership over the productive assets. This leads to ex post bargaining over the procurement of the input. In this setup, Bolton and Whinston analyze different ownership structures. The major difference to our analysis is that in Bolton and Whinston the input is transferred from the upstream unit to the downstream units by means of a bargaining process in which the owner of the upstream firm gets a share of the surplus that the input generates at the downstream level. Since downstream firms do not appropriate the entire surplus, they engage in inefficiently low ex ante investments. This inefficiency in turn creates an incentive to vertically integrate. In our model, firms are also unable to write complete ex ante contracts about the procurement of the intermediate good. In contrast to Bolton and Whinston, here the input is transferred through a spot market with flexible prices in which upstream firms have no market power.

We consider a finite set of downstream firms each of which has a stochastic requirement for a particular input—less in bad times than in good times. Downstream firms either produce the intermediate good themselves or purchase it through a Walrasian market from upstream firms with no market power. Both, up- and downstream firms have access to the same input technology. To produce the input, a firm has to build up capacity at a fixed cost. If a firm has a certain capacity level, it can produce any quantity of the input not exceeding capacity at a constant marginal cost. We assume, as is quite common in the literature (see, e.g., Williamson (1985)), that a downstream firm producing the intermediate good does not sell it. Up- and downstream firms simultaneously pick capacity levels. Nature then determines each downstream firm's input requirement. If a downstream firm's input requirement exceeds its own capacity, it shows up on the input market with positive demand. If market demand exceeds market supply, a high price prevails and vice versa so that the input market clears. Up- and downstream firms are risk neutral.

To begin with, we show that full integration always constitutes an equilibrium market structure. That is, all downstream firms have a capacity that permits to produce the maximum input requirement in good times and that is partly idle in bad times. This equilibrium, however, is inefficient. Since downstream firms do not sell the input, they cannot pool their input requirements. Pooling of capacities can only be achieved if the input is produced by upstream firms and traded in the market. For example, the situation where downstream firms do not produce the input at all and purchase their needs on the market is efficient.

Nevertheless, this efficient non-integration situation never constitutes a market equilibrium. If a downstream firm starts producing some of its own input needs, it cuts down on aggregate demand. This depresses prices in the input market so that the downstream firm gets the inframarginal units more cheaply on average. This favorable price effect outweighs the cost of idle capacity in bad times; this cost is, indeed, negligible if the vertically integrated capacity is not too large. It follows from this result that the input market will always be characterized by vertical upstream integration.

The following two questions then arise: when will we observe an efficient level of partial vertical upstream integration and under what conditions will the input market be characterized by too much vertical integration? The answer to the first question is fairly negative. If the model's parameters happen to be such that upstream firms make expected zero profits, then there exists an equilibrium with an efficient level of partial vertical upstream integration. The parameter constellation, however, is non-generic.

Our last result gives sufficient conditions for too much vertical upstream integration. If the zero profit condition fails to hold and the input requirement in good times is sufficiently high, downstream firms will have an inefficiently high level of vertically integrated capacity. The favorable effect of depressing market prices outweighs the loss from idle capacity in bad times.

We thus show that, although an intermediate good market is characterized by flexible prices and competitive upstream firms, downstream firms will always vertically integrate. Furthermore, the level of vertical upstream integration is often inefficiently high. At this point it is worth mentioning what distinguishes our approach from the literature on vertical foreclosure. In this literature (see, e.g., Hart and Tirole (1990), Ordover, Saloner, and Salop (1990), and Salinger (1988)) a downstream firm integrates upstream to foreclose its downstream rivals from input supplies. This puts the rivals in the product market at a disadvantage, allowing the downstream firm to raise its market share. In other words, in this literature the motive for vertical integration is to foreclose product market competition by raising rivals' costs (Salop–Scheffman (1988)). In our setup, downstream firms produce distinct products and, therefore, do not compete on the output markets. Accordingly, we explain vertical integration as a phenomenon that results solely from input market considerations.

The remainder is organized as follows. In the next section we describe the model. In section III we derive the efficient industry structure. The subsequent section contains our results about the equilibrium market structures. Section V concludes the paper. All proofs are relegated to the Appendix.

II. The Model

We consider the market for an intermediate good. The intermediate good is produced with a 'fixed cost' technology. First, firms have to invest in capacity having a unit cost f. Then the firm can produce the intermediate good at a constant marginal cost c, but only up to the capacity level determined in the first step. The two costs differ in the way they can be adjusted to unexpected changes in production. A firm can avoid cost c if it chooses not to produce ex post. In contrast, if a firm wants to have the capability of producing one unit ex post, it must invest f in capacity ex ante; f is sunk even if at a later stage the firm decides not to produce.

More specifically, a firm has to build up capacity $y \in \mathsf{N}_0$ at a unit cost $f > 0$.[1)] If a firm has capacity y, it can produce any quantity of the intermediate good $\upsilon \leq y,\ \upsilon \in \mathsf{N}_0$ at a constant marginal cost c. It is not possible to produce $\upsilon > y$. The technology for the intermediate good is thus given by the cost function

$$K(\upsilon, y) = \begin{cases} fy + c\upsilon, & \text{if } \upsilon \leq y; \\ \\ \infty, & \text{otherwise.} \end{cases} \tag{1}$$

For ease of exposition, we normalize marginal cost $c = 0$.

There are m upstream firms indexed by $b = 1, \ldots, m$. Upstream firms produce the intermediate product according to the technology (1). They sell it through a market to downstream firms that use the intermediate product as an input to produce their outputs. We want to make market exchange as attractive as possible for downstream firms. Therefore, our upstream sector is first of all competitive. More specifically, upstream firm b chooses capacity $y^b \in \{0, 1\},\ b = 1, \ldots, m$; upstream firms are thus 'small-scale'. Under this assumption an upstream firm cannot make positive profits by reducing its capacity. Upstream firms are either in or out of the market. They are, essentially, strategic dummies. In particular, this assumption permits the existence of equilibria in which upstream firms make expected zero profits.

Moreover, besides being competitive, we want the upstream sector to be large enough to satisfy whatever the downstream sector might need. Therefore, let m be sufficiently large (in a sense to be made more precise below) so that there are enough upstream firms to be able to produce the maximum amount of the intermediate good that might be required. To summarize the upstream sector: There is a large pool of upstream firms. Upstream firms become active on a small scale so that they have no market power. The upstream sector is thus modeled as the most favorable source of supply imaginable for downstream firms.

Let us now turn to the downstream sector using the intermediate good we consider as an input to produce their outputs. The downstream sector consists of n firms indexed by $a = 1, \ldots, n$. As in Bolton and Whinston (1993), each downstream firm produces a distinct product. There is no competition between downstream firms on their output markets. As an example, think of the intermediate product as memory chips which are used as input in the computer, the auto electronics, the machine tool, the telecommunications, the consumer electronics industries, etc.; another example of the intermediate product is the common carrier truck business which is used as an input by a plethora of downstream firms not competing on their output markets.

Each downstream firm faces stochastic demand for its output that can be either high or low. A downstream firm's requirement for the intermediate good is, therefore, also random: if output demand is low, i.e., in bad times, a downstream firm needs less of the intermediate good than in good times. To simplify the analysis, the downstream firms' input requirements are identically distributed. Formally, denote downstream firm a's, $a = 1, \ldots, n$, input requirement by

$$\tilde{x}^a = \begin{cases} \underline{x}, & \text{with probability } Pr(\underline{x}) \in (0,1); \\ \overline{x}, & \text{with probability } Pr(\overline{x}) = 1 - Pr(\underline{x}). \end{cases}$$

Let $\underline{x}, \overline{x} \in \mathsf{N}$, i.e., there is a smallest unit of account for the input and a downstream firm always needs some of the intermediate good. Furthermore, $\overline{x} > \underline{x} \geq \overline{x} - \underline{x}$. That is, measured in input terms, good times are strictly better than bad times and, more importantly, *good times are no more than twice as good as bad times.* As we will see later, the second inequality is crucial to our results. It implies that the riskless part of a downstream firm's input requirement $\underline{x}$ is so large that taking $\underline{x}$ out of market demand by vertical integration is sufficient to depress prices on the market.

Let $\pi > 0$ denote the downstream firms' reservation price per unit of the intermediate good. Accordingly, at price π a downstream firm is indifferent between obtaining and forgoing a unit of the input. As an interpretation, think of π as the price of a backstop substitute for the input, i.e., an expensive yet still profitable substitute available in abundant supply. In terms of our truck business example, think of π as the price the mail service charges for shipping goods; if we think of the input as 128 KB memory chips, 2π would be the price of a 256 KB chip.

Downstream firms can produce the intermediate good themselves according to the technology (1). Downstream firm a chooses capacity $y^a \in \mathsf{N}_0,\ a = 1, \ldots, n$. We identify a downstream firm's degree of vertical upstream integration by its capacity choice. We will call a downstream firm *non-integrated* if $y^a = 0$, *partially integrated* if $y^a \in (0, \overline{x})$, and *fully integrated* if $y^a \geq \overline{x}$.

We assume that downstream firms producing the intermediate good do not sell it. It might not be profitable for downstream firms to digress from marketing their outputs by additionally selling the input. Alternatively, the intermediate good market might be regulated such that downstream firms may produce their own requirement but are not allowed to become sellers of the input.[2)] Note that we do *not* need this assumption to establish our result that there will always be vertical integration; see also the discussion in the conclusions.

Let

$$Pr(\overline{x}) \geq f/\pi, \tag{2}$$

i.e., the probability of good times is higher than the input technology's cost-benefit ratio. More specifically, assumption (2) implies the following: If there is no market for the intermediate product, a downstream firm holds capacity $y^a = \overline{x}$ to be able to produce the maximum requirement $\overline{x}$ itself rather than forgo some input.

Let us now turn to the formulation of the game which is a simultaneous move game in capacities. Downstream firms strategically pick capacity $y^a \in \mathsf{N}_0,\ a = 1, \ldots, n$. We denote the downstream firms' vertically integrated capacities by $\mathbf{y} = (y^1, \ldots, y^n)$. All downstream firms exhaust their own input capacity and purchase the remaining requirement on the market should this be necessary.[3)] Downstream firm a's input demand is thus a random variable $\tilde{d}^a(y^a) = \max[0, \tilde{x}^a - y^a]$ with support $\underline{d}^a(y^a) := \max[0, \underline{x} - y^a]$ and $\overline{d}^a(y^a) := \max[0, \overline{x} - y^a],\ a = 1, \ldots, n$.

Denote aggregate input demand by $\tilde{D}(\mathbf{y}) = \sum_{a=1}^{n} \tilde{d}^a(y^a)$. By $D_i(\mathbf{y})$ we denote the elements of the support of aggregate input demand $\tilde{D}(\mathbf{y})$; the subscript i orders the demand realizations in increasing order of their size. With this notation the random variable describing aggregate input requirement is $\tilde{D}(\mathbf{0})$ where $\mathbf{0} = (0, \ldots, 0)$. The maximum aggregate input requirement is $D_n(\mathbf{0}) = n\overline{x}$, i.e., all n downstream firms face good times. The assumption that there are enough upstream firms to be able to produce the maximum input requirement thus means $m \geq D_n(\mathbf{0})$.

Simultaneously with downstream firms, upstream firm b chooses as a strategic variable the capacity $y^b \in \{0, 1\},\ b = 1, \ldots, m$. Call an upstream firm that picks capacity 1 active and inactive otherwise. Upstream firms tell the auctioneer their capacity $y^b,\ b = 1, \ldots, m$. Denote upstream firms' aggregate capacity by $Y = \sum_{b=1}^{m} y^b$. Call Y market capacity.

Nature then determines each downstream firm's input requirement. Downstream firms tell the auctioneer their input demand. The auctioneer clears the market by the following pricing rule

$$p = \begin{cases} 0, & \text{if } D_i(\mathbf{y}) \leq Y; \\ \pi, & \text{otherwise.} \end{cases}$$

If the market is slack, i.e., if aggregate demand does not exceed market capacity, price equals marginal production cost. At the rockbottom price $p = 0$ active upstream firms are indifferent between producing and not producing the input. Accordingly, in a buyers' market, each active upstream firm makes a loss f.

If the market is tight, i.e., if aggregate demand exceeds market capacity, price equals downstream firms' reservation price. At the sky-high price $p = \pi$ downstream firms are indifferent between obtaining and forgoing the intermediate good. In a sellers' market active upstream firms make a profit $(\pi - f)$. Note that the pricing rule clears the market. Consequently, no rationing occurs.[4)]

Up- and downstream firms are risk neutral. Upstream firm $b,\ b = 1, \ldots, m$ maximizes with respect to y^b expected profits

$$\Pi^b(\mathbf{y}, Y) = \begin{cases} \pi \sum_{D_i(\mathbf{y}) > Y} Pr\left(\tilde{D}(\mathbf{y}) = D_i(\mathbf{y})\right) - f, & \text{if } y^b = 1; \\ 0, & \text{otherwise,} \end{cases}$$

where $Y = \sum_{b=1}^{m} y^b$. As a shortcut we will write $Pr(D_i(\mathbf{y}))$ instead of $Pr\left(\tilde{D}(\mathbf{y}) = D_i(\mathbf{y})\right)$ from now on.

To determine a downstream firm's payoff function we need one more bit of notation. Consider the conditional random variables $\tilde{D}(\mathbf{y}|\tilde{x}^a = \underline{x})$ and $\tilde{D}(\mathbf{y}|\tilde{x}^a = \overline{x})$, $a = 1, \ldots, n$. We assume that for all $\mathbf{y} \in \mathsf{R}_+^n$ these random variables have full support. This is equivalent to the assumption that all the possible combinations of the individual input requirements occur with positive probability.

Downstream firm $a,\ a = 1, \ldots, n$ minimizes with respect to y^a the expected costs $C^a(\cdot)$ of obtaining the intermediate product. This expected cost is given as

$$\begin{aligned} C^a(\mathbf{y}, Y) \quad &= \quad fy^a + \pi\Big[\underline{d}^a(y^a)Pr(\underline{x})\sum\nolimits_{D_i(\mathbf{y})>Y} Pr(D_i(\mathbf{y})|\tilde{x}^a = \underline{x}) + \\ &\overline{d}^a(y^a)Pr(\overline{x})\sum\nolimits_{D_i(\mathbf{y})>Y} Pr(D_i(\mathbf{y})|\tilde{x}^a = \overline{x})\Big]. \end{aligned}$$

First, the downstream firm incurs capacity costs fy^a. Second, it pays the high price π on the market whenever the market is tight, i.e., if $D_i(\mathbf{y}) > Y$; it either pays the price on its demand $\underline{d}^a$ in bad times or on its demand $\overline{d}^a$ in good times. Downstream firms minimize with respect to y^a the expected cost $C^a(\cdot)$ of obtaining the input, $a = 1, \ldots, n$. We focus on Nash–equilibria in pure strategies of our simultaneous move game in capacities.

III. Efficient Industry Structures

Let us begin the analysis by describing the efficient industry structure for our intermediate product. Consider the allocation problem from a social planner's point of view. Recall that downstream firms do not sell the input. Accordingly, they cannot pool their input requirements.[5)] In our setup, pooling of capacities can be achieved only if the input is produced by upstream firms and traded in the market. It follows from this assumption that building up a unit of capacity in the upstream sector weakly dominates installing the capacity in the downstream sector: If a downstream firm needs the input, it gets it in either case. If a downstream firm does not need the input, the capacity idles if vertically integrated; yet when the capacity is upstream, the input can be transferred to another downstream firm. It is, therefore, efficient to install aggregate capacity entirely at the upstream level.[6)]

The planner has to find the aggregate capacity level Y that minimizes total expected costs of obtaining the intermediate product. These costs, in turn, are the cost of installing capacity plus the expected cost of purchasing the backstop substitute at price π whenever the own capacities are insufficient, i.e., if $D_i(\mathbf{0}) > Y$. The planner thus solves

$$\min_Y fY + \pi \sum\nolimits_{D_i(\mathbf{0})>Y} [D_i(\mathbf{0}) - Y] Pr(D_i(\mathbf{0})).$$

Ignoring the integer constraint, the optimal Y^* satisfies

$$f = \pi \sum_{D_i(\mathbf{0})>Y^*} Pr(D_i(\mathbf{0})). \tag{3}$$

A unit capacity costs f. The social benefit in case the capacity is used is π. The probability of the marginal unit of capacity being put to use is $\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0}))$. Accordingly, the planner sets aggregate capacity to the level where the expected benefit of the marginal unit equals its cost.

The following observations are useful for later purposes. First, if $Y^* = D_k(\mathbf{0})$ and satisfies (3), any $Y \in [D_k(\mathbf{0}), D_{k+1}(\mathbf{0})]$ gives rise to the same total costs and is, therefore, also efficient. An increase in Y leaves the right-hand side unchanged as long as $Y < D_{k+1}(\mathbf{0})$. Second, since for all $\mathbf{y}$ with $y^a \leq \underline{x}$, $a = 1, \ldots, n$, $\tilde{D}(\mathbf{y}) = \tilde{D}(\mathbf{0}) - \sum_{a=1}^{n} y^a$, the downstream capacities $\mathbf{y}$ together with the upstream capacity $Y(\mathbf{y}) := Y^* - \sum y^a$ are efficient: up- and downstream capacities add up to Y^* and $\tilde{D}(\mathbf{y})$ differs from $\tilde{D}(\mathbf{0})$ only in that the support is shifted to the left. Partially integrated capacities not exceeding $\underline{x}$ are, therefore, efficient. Partially integrated capacity in excess of $\underline{x}$ is however typically inefficient: the capacity in excess of $\underline{x}$ is used more often when it is installed at the upstream level, see section IV.

Finally, note that multiplying (3) by Y^* yields $fY^* = Y^*\pi \sum_{D_i(\mathbf{0})>Y^*} Pr(D_i(\mathbf{0}))$. Accordingly, if upstream firms provide the efficient aggregate capacity level Y^* given downstream firms are non-integrated and are compensated through the auctioneer's pricing rule, they break even. The probability that the marginal unit of capacity is put to use equals the probability of a tight market with the high price π. In the same way $Y(\mathbf{y})$ generates zero profits when downstream capacities are $\mathbf{y}$. Condition (3), therefore, guarantees upstream zero profits over a wide range of downstream capacities. As we will see later, this observation implies that upstream firms invest efficiently in capacity if the integer constraint is not binding. Accordingly, our results on vertical integration do not rely on the Williamson (1985) type of underinvestment phenomena. If the integer constraint binds, however, such an underinvestment effect additionally comes into play and reinforces the tendency to integrate.

IV. Market Structures

Let us now derive some results on the market structures for our intermediate product. First note that full integration, or equivalently no trade, is always an equilibrium market structure. More specifically, the strategy profile in which all downstream firms pick capacity

$\hat{y}^a = \overline{x},\ a = 1, \dots, n$ and all upstream firms choose capacity $\hat{y}^b = 0,\ b = 1, \dots, m$, constitutes an equilibrium.

The existence of the full integration equilibrium is an immediate consequence of assumption (2). This assumption says that the probability of good times is sufficiently high to make it worthwhile to have capacity $y^a = \overline{x}$ given that there is no input market. Capacity $\overline{x}$ is only partly used in bad times. Yet the cost of forgoing some input in good times outweighs the cost of idle capacity in bad times. Conversely, given that all downstream firms choose capacity $\overline{x}$, there is no demand for the intermediate product. Upstream firms thus never sell anything. If an upstream firm becomes active, it incurs a fixed cost without obtaining any revenue. No trade is, therefore, a Nash equilibrium for our intermediate good market. Consequently, there exists an equilibrium in pure strategies for our market which, in turn, is generated by a standard coordination failure.

Nevertheless, the full integration equilibrium is typically inefficient. Let us elaborate this point by means of an example. Suppose $n = 3,\ \underline{x} = 1,\ \overline{x} = 2,\ Pr(\underline{x}) = 1/2,\ f = 1,\ \pi = 2$, and let the downstream firms' input requirements be stochastically independent. In the full integration equilibrium downstream firms' cost $C^a(\mathbf{2}, 0) = 2,\ a = 1, 2, 3$.

Now consider the *non-integration situation* where all downstream firms have capacity $y^a = 0$ so that aggregate demand equals aggregate input requirement $\tilde{D}(\mathbf{0})$. Suppose the first 4 upstream firms are active so that market capacity is efficient, i.e., $Y = Y^* = 4$. Then, if $\tilde{D}(\mathbf{0}) \in \{D_0(\mathbf{0}) = 3,\ D_1(\mathbf{0}) = 4\},\ p = 0$ and if $\tilde{D}(\mathbf{0}) \in \{D_2(\mathbf{0}) = 5,\ D_3(\mathbf{0}) = 6\},\ p = \pi$. An active upstream firm's expected profit $\Pi^b(\mathbf{0}, 4) = 0$.

An active upstream firm has a fixed cost f. If the market is slack, $p = 0$ and active firms do not recover their overheads. Yet if the market is tight, $p = \pi$ and each active upstream firm earns a contribution margin π. If $Y = 4,\ f = \pi \sum_{D_i(\mathbf{0}) > Y} Pr(D_i(\mathbf{0}))$, i.e., the expected contribution margin equals overheads and active upstream firms make expected zero profits. Accordingly, all upstream firms are indifferent between the non-integration situation and the full integration equilibrium. A downstream firm's cost of obtaining the input in the non-integration situation is $C^a(\mathbf{0}, 4) = \pi \underline{x} 1/8 + \pi \overline{x} 3/8 = 1.75,\ a = 1, 2, 3$, i.e., all downstream firms are better off in the efficient non-integration situation than in the full integration equilibrium.

This Pareto improvement can be explained as follows. Downstream firms pay the high price π with probability $\sum_{D_i(\mathbf{0}) > Y} Pr(D_i(\mathbf{0}))$. Now suppose whenever downstream firm a has high demand $\overline{d}^a(0) = \overline{x}$ there is a sellers' market, i.e., $\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})|\tilde{x}^a = \underline{x}) = 0$. The downstream firm then always pays the high price for the large quantity, i.e., it pays $\pi\overline{x}\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})) = f\overline{x}$; accordingly, purchasing the input on the market or having capacity $y^a = \overline{x}$ amounts to the same in this case. If, however, as in our example, a tight market coincides with the low demand $\underline{d}^a(0) = \underline{x}$, i.e., $\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})|\tilde{x}^a = \underline{x}) > 0$, the downstream firm pays less than $f\overline{x}$ on the market, i.e.,

$$\pi\left[\underline{x}Pr(\underline{x})\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})|\tilde{x}^a = \underline{x}) + \overline{x}Pr(\overline{x})\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})|\tilde{x}^a = \overline{x})\right] < \pi\overline{x}\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})) = f\overline{x}.$$

The market charges the fair expected price $\pi\sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0})) = f$ to recover active upstream firms' fixed costs. If a downstream firm is lucky and has low input demand $\underline{x}$ in a sellers' market, it is strictly better off in the non-integration situation than in the full integration equilibrium.

A downstream firm has low demand in a sellers' market if market capacity $Y < D_{n-1}(\mathbf{0})$. We may, therefore, conclude that risk pooling through the market *strictly* dominates full integration if the efficient aggregate capacity Y^* satisfies (3) and $Y^* < D_{n-1}(\mathbf{0})$. These conditions follow immediately from our previous observation: If $Y^* = D_k(\mathbf{0})$ and satisfies (3), any $Y \in [D_k(\mathbf{0}), D_{k+1}(\mathbf{0})]$ gives rise to the same aggregate costs and is, accordingly, also efficient. Thus, if $Y^* = D_{n-1}(\mathbf{0})$ and satisfies (3), $Y = D_n(\mathbf{0})$ is also efficient and pooling of capacities through the market is not strictly better than full integration.

Given that the non-integration situation typically Pareto dominates the full integration equilibrium, it seems worthwhile to investigate under what conditions the efficient non-integration situation constitutes an equilibrium. In the next Proposition we will show that the non-integration situation *never* constitutes a market equilibrium. It follows from this result that in any equilibrium market structure we observe vertical upstream integration.

Proposition 1: *There does not exist an equilibrium in which downstream firms choose capacity $y^a = 0,\ a = 1, \ldots, n$.*

Let us explain this result by means of the example.[7)] If $\hat{y}^a = 0,\ a = 1, 2, 3$, an upstream firms' best response entails $\hat{Y} \in \{3, 4\}$. Suppose $\hat{Y} = 4$ so that 4 upstream firms are active. This upstream firms' best response is the most favorable one to downstream firms because it is efficient, i.e., $f = \pi \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0}))$. Active upstream firms make expected zero profits.

Suppose downstream firm 1 switches from $\hat{y}^1 = 0$ to $y^1 = 1$ so that its demand reduces from $\tilde{d}^1(0)$ to $\tilde{d}^1(1)$. Aggregate demand changes from $\tilde{D}(\mathbf{0})$ to $\tilde{D}(\mathbf{0}_1)$ where $\mathbf{0}_1 = (1, 0, 0)$. Then we have $p = 0$ if $\tilde{D}(\mathbf{0}_1) \in \{D_0(\mathbf{0}_1) = 2, D_1(\mathbf{0}_1) = 3, D_2(\mathbf{0}_1) = 4\}$ and $p = \pi$ if $\tilde{D}(\mathbf{0}_1) = D_3(\mathbf{0}_1) = 5$. The probability of a tight market thus decreases from 1/2 to 1/8. If downstream firm 1 unilaterally deviates to $y^1 = 1$, we have $C^1(\mathbf{0}_1, 4) = 1.25 < 1.75 = C^1(\mathbf{0}, 4)$. Consequently, the downstream firm is strictly better off if it picks capacity $y^1 = 1$ instead of capacity $\hat{y}^1 = 0$.

Downstream firms pay the fair expected price $f = \pi \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0}))$ on the market. A downstream firm needs $\underline{x}$ for sure and an additional $(\overline{x} - \underline{x})$ when its good times roll. If a firm purchases its entire input needs on the market, it pays $\pi \underline{x} \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})) + \pi(\overline{x} - \underline{x}) Pr(\overline{x}) \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \overline{x}) = f\underline{x} + \pi(\overline{x} - \underline{x}) Pr(\overline{x}) \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \overline{x})$. Accordingly, if the probability of a tight market coinciding with downstream firm 1's high demand were to remain unchanged by the switch to $y^1 = \underline{x}$, the firm would be indifferent between $\hat{y}^1 = 0$ and $y^1 = \underline{x}$. But this probability decreases.

Recall that $D_i(\mathbf{0}) - D_{i-1}(\mathbf{0}) = \overline{x} - \underline{x}$, $i = 1, \ldots, n$. Since $\underline{x} \geq \overline{x} - \underline{x}$, we have $D_i(\mathbf{0}_1) \leq D_{i-1}(\mathbf{0})$, $i = 1, \ldots, n$; increasing capacity makes demand when i firms face good times lower than demand was before the switch with $(i-1)$ firms having high input requirement. Therefore, $\sum_{D_i(\mathbf{0}_1)>\hat{Y}} Pr(D_i(\mathbf{0}_1)) < \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0}))$. By building up capacity $y^1 = \underline{x}$ the downstream firm cuts down aggregate demand by an amount large enough to increase the probability of a glut. Furthermore, the states of the world in which the market changes from tight to slack include contingencies where the deviating downstream firm has high demand. Since all contingencies have positive probability, $\sum_{D_i(\mathbf{0}_1)>\hat{Y}} Pr(D_i(\mathbf{0}_1)|\tilde{x}^1 = \overline{x}) < \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \overline{x})$. That is, by the switch to $y^1 = \underline{x}$ downstream firm 1 decreases the probability of a sellers' market coinciding with its own high demand. Consequently, if downstream firm 1 unilaterally deviates to $y^1 = \underline{x}$, it gets the inframarginal units $(\overline{x} - \underline{x})$ on average more cheaply. The firm is strictly better off and the non-integration situation cannot be an equilibrium.

We have thus shown that we will *always* observe vertical upstream integration. A downstream firm building up capacity has to take into account the following effects: The first effect is that it costs f to build up a unit of capacity for which the market charges the expected price $\pi \sum_{D_i > Y} Pr(D_i)$; this effect is thus $f - \pi \sum_{D_i > Y} Pr(D_i)$. The second effect is that f is lost whenever there is idle capacity; in the market the capacity could be obtained only when it is needed. The third (oligopsony) effect is that the downstream firm cuts down on aggregate demand and may thus decrease the probability that high prices will prevail on the market. Accordingly, the downstream firm may gain from decreasing the expected price of all inframarginal units by withdrawing a marginal unit from the spot market.

Now consider a downstream firm switching from capacity $y = 0$ to capacity $y = \underline{x}$. If upstream firms make expected zero profits given that all downstream firms pick capacity 0, the first effect is zero. The second effect is zero because the downstream firm needs $\underline{x}$ for sure. Yet, the third effect is favorable to the downstream firm. The switch to $\underline{x}$ decreases aggregate input demand by an amount large enough to decrease the probability of high prices prevailing on the input market; in expectation the downstream firm pays less for its inframarginal units $(\overline{x} - \underline{x})$. Overall then, the oligopsony effect makes it attractive for downstream firms to build up positive capacity.

Note that the downstream firms' incentive to build up capacity is strict. Accordingly, our non-existence result still holds if downstream firms have a 'slightly worse' technology than upstream firms, i.e., if, for example, downstream firms' fixed cost is $f' := f + \epsilon$ with $\epsilon > 0$ and small. However, if downstream firms' fixed cost is f', any vertical upstream integration is inefficient. Upstream firms can build up the same capacity at a lower cost. Thus, in this case we may immediately conclude from Proposition 1 that we observe inefficient vertical upstream integration on the intermediate good market.

If upstream firms make positive expected profits, it is cheaper to build up own capacity than purchasing it on the market. The first effect is thus favorable to the downstream firm. In this case aggregate capacity Y is so low that $f < \pi \sum_{D_i(\mathbf{0}) > Y} Pr(D_i(\mathbf{0}))$ and an additional Williamson type of underinvestment phenomenon comes into play. Then the following Corollary to Proposition 1 holds:

Corollary: *There does not exist an equilibrium in which upstream firms make positive profits and any downstream firm has capacity* $y^a < \underline{x}$, $a = 1, \ldots, n$.

If a downstream firm has a capacity of less than $\underline{x}$, it must always go to the market to obtain the good. If upstream firms make positive profits, the market charges more than $f(\underline{x} - y^a)$ for the downstream firm's certain demand — the amount the downstream firm incurs if it produces its certain demand $(\underline{x} - y^a)$ itself.

So far we have shown that there will always be vertical integration. Yet, as we have seen in the previous section, partial integration need not be inefficient. So the following two questions naturally arise: will we ever observe an efficient level of vertical upstream integration and under what conditions will the input market be characterized by too much vertical integration?

Even though the answer to the first question is fairly negative, it is useful to discuss it for later purposes. The following can be shown. Suppose the integer constraint in the planner's problem does not bind so that some $Y^* = D_k(\mathbf{0})$ solves (3). Then the following partial integration situation is an equilibrium: All downstream firms choose capacity $\underline{x}$ so that downstream capacities are $\underline{\mathbf{x}} = (\underline{x}, \ldots, \underline{x})$. That is, downstream firms pool the risky part of their input requirements. Upstream capacity is $Y(\underline{\mathbf{x}}) = (Y^* - n\underline{x})$ with which upstream firms make zero profits. Since up- and downstream capacities add up to Y^*, this partial integration equilibrium is efficient.

Why does a downstream firm not deviate by increasing its own capacity above $\underline{x}$? With such an increase the first of our three effects is zero because upstream firms make zero profits. The second effect is unfavorable since the additional capacity is idle in bad times. The third effect, finally, is zero. If the downstream wishes to increase the probability of a glut, it must deviate to $\overline{x}$. But then it does not gain from the lower expected price because it does not have any inframarginal units left from which to benefit.

It is of some interest to discuss why (3), which implies zero profits in the upstream sector, is necessary to support this equilibrium. First, if upstream firms make positive expected profits, the first effect is favorable, making an increase in capacity attractive. The second reason is more subtle. $Y(\underline{\mathbf{x}})$ is the *minimum* upstream capacity that generates zero profits. Since Y^* satisfies (3), any capacity $Y \in [Y(\underline{\mathbf{x}}), Y(\underline{\mathbf{x}}) + \underline{x})$ is efficient and, moreover, generates zero profits. Accordingly, upstream firms are indifferent between these capacity levels. Yet, for an efficient partial upstream equilibrium to exist, upstream firms have to pick $Y(\underline{\mathbf{x}})$, i.e., the lowest market capacity that allows zero profits. With this upstream capacity, a small decrease in market demand does not decrease the probability that high prices prevail. The incentive for downstream firms to cut down on aggregate demand to decrease the probability of a sellers' market is, therefore, zero at the margin. In contrast, if upstream firms pick the highest market capacity that allows zero profits, a small decrease in market demand decreases the probability of a tight market. Accordingly, the downstream firms' incentive to cut down on aggregate demand at the margin is strictly positive in this case. When the zero profit premise fails to hold (i.e., if $f < \pi \sum_{D_i > Y} Pr(D_i)$, for $Y < D_k$ and the inequality is reversed for $Y \geq D_k$) upstream firms will build up the highest market capacity $(D_k - 1)$ that still allows positive profits. Consequently, the expected zero profit assumption is necessary to eliminate the downstream firms' incentive to cut down on aggregate demand at the margin.

Nevertheless, the zero profit condition holds only for non-generic parameter values. What can be said about the efficiency properties of market structures if the zero profit condition fails to hold? When will downstream firms have inefficiently high capacity, i.e., a capacity level in excess of $\underline{x}$?[8)] We have just seen that if the zero profit condition fails to hold, upstream firms build up the highest market capacity that allows positive profits. A small decrease of aggregate demand is thus sufficient to decrease the probability of a sellers' market. Accordingly, on the one hand, a downstream firm is tempted to increase its capacity slightly beyond $\underline{x}$ to cut down on aggregate demand and save on the inframarginal units. On the other hand, if a downstream firm has capacity in excess of $\underline{x}$, it has idle capacity in tough times. In the next Proposition we provide a sufficient condition for the third (favorable) effect to outweigh the second (unfavorable) effect so that we observe too much vertical integration.

Proposition 2: *If there exists no $Y^* \in \mathsf{N}$ satisfying* (3) *and $\overline{x}$ is sufficiently large, then there does not exist an equilibrium in which downstream firms choose capacity $y^a \in [0, \underline{x}],\ y \in \mathsf{N}_0,\ a = 1, \ldots, n$.*

We will thus observe an inefficiently high level of vertical upstream integration if upstream firms make positive expected profits and $\overline{x}$ is large. For $y^a < \underline{x}$ this 'too much vertical integration' result follows from the Corollary. Therefore, consider the situation where all downstream firms pick capacity $y^a = \underline{x}$, i.e., where they pool the risky part of their input requirements in the market. Upstream firms then have the highest market capacity that still allows for positive profits. If market capacity increases by one unit, all active upstream firms make expected losses. This in turn implies that a downstream firm's switch from $y^a = \underline{x}$ to $y^a = \underline{x} + 1$ is sufficient to increase the probability of a slack market or, equivalently, to lower the expected price.

In terms of our three effects such a deviation implies the following. The first effect is strictly favorable to a downstream firm. The market charges an expected price that is too high to recover upstream firms' overheads. The second effect is strictly unfavorable to a downstream firm. The downstream firm has capacity in excess of $\underline{x}$. The excess capacity is idle in bad times. The third effect, however, is strictly favorable. By the switch to $y^a = \underline{x}+1$, a downstream firm decreases the probability of a tight market. This means that it gets the inframarginal units $(\overline{x} - \underline{x} - 1)$ more cheaply on average. If $\overline{x}$ is sufficiently large, the third effect outweighs the second effect. Consequently, a downstream firm will deviate to the inefficiently high capacity level $y^a = \underline{x} + 1$.

V. Conclusions

We have shown that, although an input market is characterized by flexible prices and competitive upstream firms, downstream firms will always vertically integrate. If a downstream firm starts producing some of its own input needs, it cuts down on aggregate input demand thus depressing prices in the market. This favorable oligopsony effect outweighs the risk of idle capacity in bad times given that the vertically integrated capacity is not too large. This result does not hinge on the assumption that downstream firms do not sell the input. Moreover, we have shown that the incentive to depress prices on the input market frequently leads to inefficiently high levels of vertically integrated capacity.[9)]

If downstream firms are allowed to sell the input, our results change as follows. The cost of vertical integration is reduced because a downstream firm can sell input for which it has no use; the cost is not eliminated since there may be contingencies in which the market price is zero. The oligopsony effect we have identified is still at work. The possibility of sale, therefore, increases the incentive to integrate. With sale, however, vertical integration is not inefficient. Accordingly, if we allow downstream firms to sell the input, we retain the robust result that the oligopsony effect will push the industry away from vertical disintegration.

To conclude, let us compare our 'too much vertical integration' result with the textbook monopsonist who is the sole buyer of an input supplied by competitive firms but subject to a rising supply price. Because of the rising supply price, the monopsonist's expenditure for an additional input unit exceeds the supply price. Therefore, the monopsonist uses too little of the input. Although this outcome is socially inefficient, the monopsonist is better off than if he were a price-taker. Moreover, upstream integration may eliminate this monopsony inefficiency and increase the monopsonist's profits (McGee and Basset (1976)). Accordingly, the textbook monopsonist is happy about his market power. In contrast, in our model non-integration is not only efficient, but also downstream firms typically do better if they completely rely on the market rather than having their own capacities. A ban on vertical upstream integration, which implements the efficient non-integration situation as the equilibrium industry structure, will, therefore, be in the downstream firms' interest.

Endnotes

1) By restricting the capacity choices to be integers we, essentially, make the players' strategy spaces finite.

2) See Carlton (1979) for a more elaborate discussion of the first argument. The Motor Carrier Act of 1935 created the conditions given in the second argument. A license is needed to enter the common carrier truck business. Private carriage, i.e., carriage in trucks owned and operated by the shipper are exempted from regulation. See Kahn (1971, pp. 14-21).

3) It follows from assumption (2) that this behavior is indeed optimal. Withholding input demand ex post costs $Pr(\overline{x})\pi$ in expected terms; if a downstream firm builds up capacity ex ante and leaves it idle in bad times, it incurs expected costs f. Accordingly, if a downstream firm wants to manipulate the market, own capacities are a better means than withholding demand.

4) If $D_i(\mathbf{y}) = Y$, any price $p \in [0, \pi]$ clears the market. The reason why we choose $p = 0$ is as follows. Suppose up- and downstream firms' capacity choices $\mathbf{y}$ are such that $D_i(\mathbf{y}) = Y$ with positive probability and, moreover, that $p > 0$ in this case. If, say, downstream firm 1 increases its capacity by a small amount, it reduces aggregate demand by the same quantity. Let $\mathbf{y}'$ be the new vector of capacities. The downstream firm turns the event $D_i(\mathbf{y}) = Y$ and $p > 0$ into an event $D_i(\mathbf{y}') < Y$ and $p = 0$. Accordingly, the downstream firm gains $p > 0$ with positive probability. To rule out this incentive to cut down on aggregate demand at the margin, we set $p = 0$ if $D_i(\mathbf{y}) = Y$. Technically, this assumption guarantees upper hemi-continuity of the downstream firms' best-reply correspondences over the set of mixed strategies.

5) Note that we rule out horizontal mergers. If downstream firms merge horizontally, they can pool their input requirements.

6) In section IV we provide conditions ensuring that risk pooling through the market *strictly* dominates vertical integration.

7) It is straightforward to show that in fact any capacity choices $\mathbf{y}$ with $y^a \in [0, 2\underline{x} - \overline{x}]$, $y^a \in \mathsf{N}_0$, $a = 1, \ldots, n$ cannot be an equilibrium.

8) Recall that capacity in excess of $\underline{x}$ is inefficient if and only if $Y^* < D_{n-1}(\mathbf{0})$.

9) Emons (1994) uses a similar oligopsony effect to generate an inefficiently low level of trade in an international economics context.

Appendix

Proof of Proposition 1: Suppose not. Let $\overline{Y} := \max\{Y \in \mathsf{N}_0 | f \leq \pi \sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0}))\}$. An upstream firms' equilibrium aggregate capacity $\hat{Y}$ satisfies $\hat{Y} \leq \overline{Y}$. If $Y > \overline{Y}$, $Y \in \mathsf{N}$, $f > \pi \sum_{D_i(\mathbf{0})>Y} Pr(D_i(\mathbf{0}))$; all active upstream firms make expected losses and are better off if they become inactive.

Consider the case where $f < \pi(1 - Pr(D_0(\mathbf{0})))$ so that $\hat{Y} > D_0(\mathbf{0})$; that is, there is a buyers' market with positive probability (the converse case is ruled out by assumption (2)). Suppose downstream firm 1 deviates to $y^1 = \underline{x}$ while $\hat{y}^a = 0,\ a = 2, \ldots, n$. Downstream firm 1's input demand thus reduces from $\tilde{d}^1(0)$ to $\tilde{d}^1(\underline{x})$ and aggregate input demand from $\tilde{D}(\mathbf{0})$ to $\tilde{D}(\mathbf{0}_{\underline{x}})$ where $\mathbf{0}_{\underline{x}} = (\underline{x}, 0, \ldots, 0)$.

Since $\overline{x} - \underline{x} \leq \underline{x}$, we have $D_i(\mathbf{0}_{\underline{x}}) \leq D_{i-1}(\mathbf{0}),\ i = 1, \ldots, n$; increasing capacity makes demand with i firms facing good times lower than demand was before the switch with $(i-1)$ firms having high input requirement. Thus, $\sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})) > \sum_{D_i(\mathbf{0}_{\underline{x}})>\hat{Y}} Pr(D_i(\mathbf{0}_{\underline{x}}))$. That is, if downstream firm 1 switches to $y^1 = \underline{x}$ there is more often a glut on the input market than if it sticks to $\hat{y}^1 = 0$. Moreover, full support of $\tilde{D}(\mathbf{y}|\tilde{x}^1 = \underline{x})$ and $\tilde{D}(\mathbf{y}|\tilde{x}^1 = \overline{x})$ implies $\sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \overline{x}) > \sum_{D_i(\mathbf{0}_{\underline{x}})>\hat{Y}} Pr(D_i(\mathbf{0}_{\underline{x}})|\tilde{x}^1 = \overline{x})$ Thus,

$$
\begin{aligned}
&C^1(\mathbf{0}, \hat{Y}) = \\
&\pi\underline{x}Pr(\underline{x}) \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \underline{x}) + \pi\overline{x}Pr(\overline{x}) \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \overline{x}) \geq \\
&f\underline{x} + \pi(\overline{x} - \underline{x})\pi\overline{x}Pr(\overline{x}) \sum_{D_i(\mathbf{0})>\hat{Y}} Pr(D_i(\mathbf{0})|\tilde{x}^1 = \overline{x}) > \\
&f\underline{x} + \pi(\overline{x} - \underline{x})\pi\overline{x}Pr(\overline{x}) \sum_{D_i(\mathbf{0}_{\underline{x}})>\hat{Y}} Pr(D_i(\mathbf{0}_{\underline{x}})|\tilde{x}^1 = \overline{x}) > C^1(\mathbf{0}_{\underline{x}}, \hat{Y})
\end{aligned}
$$

where the first inequality follows from algebraic manipulations and the observation that $\hat{Y} \leq \overline{Y}$. Consequently, downstream firm 1 is strictly better off if it picks $y^1 = \underline{x}$ instead of $\hat{y}^1 = 0$.

Q.E.D.

Proof of Proposition 2: First note that the non-existence of $Y^* \in \mathsf{N}$ satisfying (3) implies the non-existence of $Y^-(\mathbf{y}) := \min\{Y \in \mathsf{N}_0 | \pi \sum_{D_i(\mathbf{y})>Y} Pr(D_i(\mathbf{y})) = f\}\ \forall \mathbf{y} = (y^1, \ldots, y^n)$ with $y^a \in [0, \overline{x}],\ a = 1, \ldots, n$. Suppose on the contrary that there exists $Y^-(\mathbf{y})$ for some $\mathbf{y}$. Since $\tilde{D}(\mathbf{y}) = \tilde{D}(\mathbf{0}) - \sum_{a=1}^n y^a,\ Y' := Y^-(\mathbf{y}) + \sum_{a=1}^n y^a$ satisfies $\pi \sum_{D_i(\mathbf{0})>Y'} Pr(D_i(\mathbf{0})) = f$, contradicting the non-existence of Y^*. For $y^a < \underline{x}$ the Proposition follows from the Corollary.

It remains to be shown that $\underline{\mathbf{x}} = (\underline{x},, \ldots, \underline{x})$ cannot be an equilibrium. Suppose not. Let $Y^+ := \max\{Y \in \mathsf{N}_0 | f < \pi \sum_{D_i(\underline{\mathbf{x}})>Y} Pr(D_i(\underline{\mathbf{x}}))\}$. The non-existence of Y^* implies that in any equilibrium the upstream capacity $\hat{Y} = Y^+$. If $Y > Y^+$, $Y \in \mathsf{N}$, $f > \pi \sum_{D_i(\underline{\mathbf{x}})>Y} Pr(D_i(\underline{\mathbf{x}}))$; all active upstream firms make expected losses and are better off if they become inactive.

Consider the case where $f < \pi(1 - Pr(D_0(\underline{\mathbf{x}})))$ so that $\hat{Y} > D_0(\underline{\mathbf{x}}) = 0$. That is, we observe a buyers' market with positive probability (the converse case is ruled out by assumption (2)). Suppose downstream firm 1 switches from $y^1 = \underline{x}$ to $y^1 = \underline{x}+1$ so that aggregate input demand changes to $\tilde{D}(\underline{\mathbf{x}}_1)$ where $\underline{\mathbf{x}}_1 = (\underline{x}+1, \underline{x}, \ldots, \underline{x})$. Then we have $\sum_{D_i(\underline{\mathbf{x}})>\hat{Y}} Pr(D_i(\underline{\mathbf{x}})) > \sum_{D_i(\underline{\mathbf{x}}_1)>\hat{Y}} Pr(D(\underline{\mathbf{x}}_1))$ and $\sum_{D_i(\underline{\mathbf{x}})>\hat{Y}} Pr(D_i(\underline{\mathbf{x}})|\tilde{x}^1 = \overline{x}) > \sum_{D_i(\underline{\mathbf{x}}_1)>\hat{Y}} Pr(D_i(\underline{\mathbf{x}}_1)|\tilde{x}^1 = \overline{x})$. Upstream firms have the highest possible capacity that allows positive profits. Increasing downstream firms' capacity by one unit is thus sufficient to decrease the probability of a sellers' market.

We then have

$$C^1(\underline{\mathbf{x}}_1, \hat{Y}) = f(\underline{x}+1) + \pi(\overline{x} - \underline{x} - 1)Pr(\overline{x}) \sum_{D_i(\underline{\mathbf{x}}_1)>\hat{Y}} Pr(D_i(\underline{\mathbf{x}}_1)|\tilde{x}^1 = \overline{x}) \leq$$
$$f\underline{x} + \pi(\overline{x} - \underline{x})Pr(\overline{x}) \sum_{D_i(\underline{\mathbf{x}}_1)>\hat{Y}} Pr(D_i(\underline{\mathbf{x}}_1)|\tilde{x}^1 = \overline{x}) +$$
$$\pi\Big(f - Pr(\overline{x}) \sum_{D_i(\underline{\mathbf{x}}_1)>\hat{Y}} Pr(D_i(\underline{\mathbf{x}}_1)|\tilde{x}^1 = \overline{x})\Big) < C^1(\underline{\mathbf{x}}, \hat{Y}),$$

where the last inequality holds if $\overline{x}$ is sufficiently large.

Q.E.D.

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