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Efficient Liability Rules for an Economy with Non-identical Individuals*

by

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Abstract

This paper is about accident situations involving two risk-neutral parties. Both parties engage in activities that are profitable but affect the magnitude of possible bilateral accidents. We analyse how the activity choices can be decentralized by liability rules that assign the costs to the two parties to an accident. We show that rules which share damages are superior to rules where one party bears the entire accident costs.

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I. Introduction

This paper is about accident situations involving two risk-neutral parties. Both parties, injurers and victims, engage in activity levels that are profitable but affect the magnitude of possible bilateral accidents. We analyse how the injurers' and victims' activity choices can be decentralized by liability rules that assign the accident costs to the two parties. We show that rules which share damages are superior to negligence rules where one party bears the entire accident costs.

Accidents involving more than one party are an example of externalities. By their choices of activity levels, two parties jointly determine the magnitude of a possible accident. Without any legal rules each party bears some fraction of the accident costs. Individually rational behaviour then typically leads to an inefficient resource allocation. Each agent takes into account the effects of his choice on the fraction of the accident costs he incurs. He ignores the effects of his choice on the other individuals in the economy.

Tort liability may be seen an incentive device whose goal is to induce an efficient pattern of individual behaviour. A liability rule allocates the entire actual damages to the two parties to an accident according to the activity levels they have chosen. If the law prescribes a liability scheme, agents base their choices on the costs they incur under the rule. Accordingly, a liability rule is a mechanism that decentralizes behaviour.

The problem of designing efficient liability rules has mainly been studied in models where all injurers and all victims are identical (see, e.g., Brown (1973)). In this special case efficiency requires that all injurers engage in the same activity level as well as all victims. The efficient allocation can be decentralized by a negligence rule. Such a rule holds injurers not liable if they meet the efficient due activity level; otherwise, injurers are strictly liable. In order not to be liable, injurers engage in the efficient due activity level. Since victims bear the entire accident costs, they also choose the efficient level.

If injurers (victims) are different from each other, efficiency requires that different individuals engage in different activity levels. In this more general setup, the efficient allocation can be implemented by a negligence rule using the incremental Learned Hand formulation (see, e.g., Posner (1986 p. 151-152)). Under this rule, an injurer is not liable if his marginal utility from the activity is greater than or equal the marginal damage that he causes; otherwise, the injurer is strictly liable.

The negligence rule using the incremental Learned Hand formulation raises two problems. First, it requires that courts can ascertain individual preferences, a task that is typically difficult to accomplish. Once we drop the assumption that courts can ascertain utility functions, we are bound to find liability rules that depend only on the activity choices of the two parties to an accident and not on their individual preferences.

Even if courts are able to discern preferences, a second problem remains. The negligence rule using the incremental Learned Hand formulation implicitly defines a personalized due activity level for each injurer and thus raises problems with respect to the equal protection clause. Think as an example of the injurers' activity level as the speed at which car drivers travel. We might wish to treat a medical doctor on his urgent way to a patient different from a tourist if both exceed the speed limit. Yet, are we willing to accept, e.g., income dependent speed limits that allow yuppies to drive faster than the unemployed? If we interpret the equal protection clause such that a law should treat everybody who does the same thing in the same way, we are again restricted to use liability rules that are only conditional on the activity choices.

A liability rule that depends only on the activity choices of the two parties to an accident will in general not implement the efficient allocation. The problem is then to find liability rules that decentralize second-best allocations. The literature suggests (see, e.g., Shavell (1987 p. 86-89)) negligence rules with an average due activity level, the so called 'reasonable man standard', be applied. These rules induce all injurers to pick the same 'reasonable' activity level. This leads to an inefficiency because some injurers engage in too high while others engage in too low activity. Yet, since injurers are not liable, victims bear the entire accident costs and choose activity levels that are efficient given the injurers' inefficient choice.

Negligence rules that depend only on the activity levels and not on individual preferences are thus an apt device in creating incentives for the parties to an accident. In the identical individuals world, they exhibit nice efficiency properties as long as risk-sharing aspects are set aside, as we will do throughout the paper. They are deemed superior to any liability rule that shares an actual damage between injurer and victim. Landes and Posner (1987 p.82-84, p.314) even call the recent U.S. movements towards sharing rules 'anomalies' that can not be justified on efficiency grounds.

In this paper we show that liability rules that exhibit sharing features are superior to negligence rules if individuals are not identical. In our setup individuals differ in the marginal utility they obtain from the activity. In an efficient allocation an individual with low marginal utility engages in a lower activity level than an individual with high marginal utility. We construct a liability scheme where the fraction of the loss injurers bear increases with their activity choice. That is, there is some range where accident costs are shared. This increasing liability scheme induces injurers with low marginal utility to engage in a lower activity level than injurers with high marginal utility. This increases welfare relative to a negligence rule where all injurers pick in the same activity level. Under our liability rule victims no longer bear the entire accident costs. Therefore, they will increase their activity choices relative to the negligence rule. Yet the rule is constructed such that this increase in the victims' activity choices has no effect on overall welfare.

We thus show that as soon as we leave the identical injurer/identical victim world, negligence rules are no longer efficient in providing the correct incentives. A liability rule that shares the accident costs between the two parties implements a superior allocation of the activity levels. We furthermore fully characterize the optimal liability rule for the case where damages are additively separable in the activity levels.

The paper is organized as follows. Section II outlines the model and derives the efficient allocation of the activity levels. In section III we describe how liability rules decentralize the activity choice and review the literature in some more detail. In section IV we derive some properties of second-best optimal liability rules.

II. The Model and the First-best Optimum

Consider an economy where accidents involving two kinds of parties, injurers and victims, may occur. Each injurer $i, i = 1, ..., n$, engages in an activity level $x \in [\underline{x}, \bar{x}], \underline{x} > 0$. An activity level x generates monetary utility $U_i(x)$ for injurer i with $U_i'(x) > 0$ and $U_i''(x) < 0$. Injurers differ in the marginal utility they obtain from the activity. Let injurers be ordered such that

$$U_i'(x) < U_{i+1}'(x) \quad \forall x \in [\underline{x}, \bar{x}], \quad i = 1, ..., n-1, \tag{1}$$

meaning that for all possible values of x injurer i has lower marginal utility than injurer $i+1$.

Victim $j, j = 1, ..., n$, chooses an activity level $y \in [\underline{y}, \bar{y}], \underline{y} > 0$ that yields monetary utility $V_j(y)$ with $V_j'(y) > 0$ and $V_j''(y) < 0$.[1] Victims are also ordered according to their marginal utility from the activity, meaning that

$$V_j'(y) < V_{j+1}'(y) \quad \forall y \in [\underline{y}, \bar{y}], \quad j = 1, ..., n-1. \tag{2}$$

Let $H(x, y)$ denote the expected monetary harm from an accident involving an injurer at the activity level x and a victim at the activity level y. The quantity $H(\cdot)$ takes into account the effects activity levels have on both the probability and the magnitude of an accident. That is, $H(x, y) \equiv p(x, y)h(x, y)$ where $p(x, y)$ denotes the probability of an accident and $h(x, y)$ the actual loss given activity levels x and y. We assume that damages depend only on the activity choices of the parties involved in an accident and not on other characteristics of the victim or injurer. Let $H_x > 0, H_{xx} \geq 0, H_y > 0, H_{yy} \geq 0$, and $H_{xy} \geq 0$. Expected damages are increasing in both parties' activity levels at a non-decreasing rate. The expected harm associated with injurer i is $\sum_j H(x_i, y_j)$ and with victim j accordingly $\sum_i H(x_i, y_j)$. Note that although we refer to the two parties as injurers and victims, this does not necessarily imply that victims suffer from the loss in the absence of legal rules. Injurers, a third party, even any combination of all individuals in the society may suffer from the loss. The harm $H(\cdot)$ constitutes a loss for the economy we analyse.[2]

Denote the injurers' activity levels by $X = (x_1, ..., x_n)$ and the victims' activity levels by $Y = (y_1, ..., y_n)$. Social (utilitarian) welfare is taken to be the sum of the monetary utilities minus the expected losses,

$$W(X, Y) = \sum_i U_i(x_i) + \sum_j V_j(y_j) - \sum_i \sum_j H(x_i, y_j). \tag{3}$$

We assume that $W(\cdot)$ has an interior maximizer (X^*, Y^*) that satisfies the first-order conditions[3]

$$U_i'(x_i^*) = \sum_j H_x(x_i^*, y_j^*) \quad i = 1, ..., n \quad \text{and}$$

$$V_j'(y_j^*) = \sum_i H_y(x_i^*, y_j^*) \quad j = 1, ..., n.$$

In the social optimum, each individual's marginal utility from the activity equals the marginal expected damage it thereby causes. Since $U_i' < U_{i+1}' \quad (V_j' < V_{j+1}')$, we have that $x_i^* < x_{i+1}^*, i = 1, ..., n-1 \quad (y_j^* < y_{j+1}^*, j = 1, ..., n-1)$. Efficiency requires that injurer i (victim j) engages in a lower activity level than injurer $i+1$ (victim $j+1$).

As an example think of injurers as car drivers and of victims as bicyclists. Both parties choose the speed at which they travel. Drivers and bicyclists like to speed. The faster both parties go, the higher is the damage in case of an accident. Economic efficiency requires that drivers (bicyclists) with low marginal utility travel at a lower speed than drivers (bicyclists) with high marginal utility.[4]

In the absence of legal rules each individual maximizes the utility out of its respective activity level minus the expected accident costs it has to bear. If the bicyclists incur the entire damages, all car drivers obviously go as fast as possible. If a third party suffers from the loss, both all drivers and all bicyclists travel at maximum speed. Consequently, there is typically a need for a mechanism that decentralizes the activity choices in an efficient way.

III. Decentralizing the Activity Choice by Liability Rules

Let us now analyse how allocations of the activity levels (X, Y) may be decentralized by liability rules. A liability rule prescribes which part of an actual loss has to be borne by the injurer and the victim. We will identify a liability rule by the part of the expected loss $l(x, y)$ the injurer bears, with $0 \leq l(x, y) \leq H(x, y)$. Actual damage payments are thus the amount $l(x, y)$ divided by the probability of an accident $p(x, y)$ given x and y. Under the liability scheme $l(\cdot)$ the victim bears $H(x, y) - l(x, y)$. Accordingly, a loss is entirely split up between the injurer and the victim and we do not allow for punitive damages.

We say that the liability rule $l(\cdot)$ implements the activity levels $(\tilde{X}, \tilde{Y}) = \big((\tilde{x}_1, ..., \tilde{x}_n), (\tilde{y}_1, ..., \tilde{y}_n)\big)$ if and only if $(\tilde{X}, \tilde{Y})$ is a Nash equilibrium to the game in which each party simultaneously picks activity levels and pays damages according to the rule $l(\cdot)$. Formally, $(\tilde{X}, \tilde{Y})$ satisfy

$$U_i(\tilde{x}_i) - \sum_j l(\tilde{x}_i, \tilde{y}_j) \geq U_i(x) - \sum_j l(x, \tilde{y}_j) \quad \forall x \in [\underline{x}, \bar{x}], \quad i = 1, ..., n \quad \text{and}$$

$$V_j(\tilde{y}_j) - \sum_i [H(\tilde{x}_i, \tilde{y}_j) - l(\tilde{x}_i, \tilde{y}_j)] \geq V_j(y) - \sum_i [H(\tilde{x}_i, y) - l(\tilde{x}_i, y)]$$
$$\forall y \in [\underline{y}, \bar{y}], \quad j = 1, ..., n.$$

Given, e.g., the victims' choice of $\tilde{Y}$, each injurer chooses x so as to maximize the utility out of his activity minus the expected payments he incurs under the rule $l(\cdot)$.[5]

Most of the literature (see, e.g., Brown (1973), Shavell (1980)) considers the special case where all injurers are identical as well as all victims. Maximizing the social welfare function (3) in this setup yields $x_i^* = x^*, \quad i = 1, ..., n$ and $y_j^* = y^*, \quad j = 1, ..., n$. Efficiency requires that all injurers pick the same activity level as well as all victims.

One of the important propositions from this literature is that the social optimum can be decentralized by the negligence rule

$$\hat{l}(x, y) = \begin{cases} 0, & \text{if } x \leq x^*; \\ H(x, y), & \text{otherwise.} \end{cases}$$

Under this negligence rule injurers are not liable if their activity level does not exceed x^*. If injurers choose some $x > x^*$, they are strictly liable, i.e., they have to pay for the entire damage. Suppose victims engage in Y^*. Given the discontinuous negligence rule, it is an optimal strategy for injurers to choose x^*, generating the highest utility among the activity levels that involve no damage payments. Given the injurers' behaviour, victims bear, willy-nilly, the entire accident costs. Since they take into account all of the adverse effects of their behaviour which means they maximize $V(y) - nH(x^*, y)$, they choose the socially optimal level y^*. Thus, the negligence rule with a due activity level x^* decentralizes the social optimum in this identical injurer/identical victim economy.

Analogously, the first-best optimum can be decentralized by the dual rule of strict liability with contributory negligence

$$l(x,y) = \begin{cases} H(x,y), & \text{if } y \leq y^*; \\ 0, & \text{otherwise.} \end{cases}$$

The injurer is liable unless the victim is found negligent. Since both rules are symmetric, in what follows we restrict our attention to the negligence rule. Nevertheless, all of our results also apply to the rule of strict liability with contributory negligence that defines a due activity level for victims.

Let us now return to our more interesting economy with non-identical individuals. In this case, the first-best optimum (X^*, Y^*) can be decentralized by the following negligence rule using the incremental Learned Hand formulation (see, e.g., Posner (1986 p. 151-152))

$$1\big(U_1'(x), ..., U_n'(x), H_x(x, y_1^*), ..., H_x(x, y_n^*), x, y\big) = \begin{cases} 0, & \text{if } U_i'(x) \geq \sum_j H_x(x, y_j^*); \\ H(x,y), & \text{otherwise.} \end{cases}$$

In case of an accident, an injurer is not deemed liable if his marginal utility from the activity is greater than or equal to the marginal expected damage he thereby causes; otherwise, the injurer is strictly liable. It is straightforward to check that under this rule it is an optimal strategy for injurer i to choose his socially efficient level x_i^* given that victims engage in Y^*.

The negligence rule using the incremental Learned Hand formulation requires that in case of an accident a court can ascertain each injurer's utility function. Once we drop the assumption that courts can ascertain each person's utility function, we are restricted to use liability rules of the form $l(x, y)$ that only depend on the measurable injurers' and victims' activity levels. These rules will in general not implement the first-best optimum so that we are restricted to find a liability scheme that implements a second-best allocation of the activity levels. Yet note that any liability rule of the form $l(x, y)$ equally applies to all individuals and thus raises no problems regarding the equal protection clause.

In this situation of incomplete information, the literature suggests (see, e.g., Diamond (1974), Posner (1986 p. 151-152), Shavell (1987 p. 86-89)) and courts tend to apply a negligence rule using the reasonable man standard. Take some average (in legal parlance 'reasonable') injurer and determine his socially optimal activity level that we will denote by $\hat{x}$. Note that $x_1^* < \hat{x} < x_n^*$. Define $\hat{x}$ as the due activity level of the negligence rule

$$\hat{l}(x, y) = \begin{cases} 0, & \text{if } x \leq \hat{x}; \\ H(x, y), & \text{otherwise.} \end{cases}$$

If this rule works in the desired way, all injurers choose the due activity level $\hat{x}$, i.e., the injurers' activity levels are given as $\hat{X} = (\hat{x}, ..., \hat{x})$.[6] This causes an inefficiency because injurers below the average engage in too high whereas injurers above the average engage in too low activity compared to the social optimum. Since all injurers pick $\hat{x}$, victims bear the entire accident costs and engage in activity levels $\hat{Y} = (\hat{y}_1, ..., \hat{y}_n)$ that are efficient given the injurers' inefficient choice.

In what follows we will study the problem of designing second-best optimal liability rules in some more detail. We will show, among other things, that a negligence rule using the reasonable man standard is not optimal.[7]

IV. Properties of Second-best Optimal Liability Rules

In this section we will derive some properties of second-best optimal liability rules. We will first confine our attention to liability rules that induce allocations of the form $(\hat{X}, \hat{Y}) = \big((\hat{x}, ..., \hat{x}), (\hat{y}_1, ..., \hat{y}_n)\big)$, i.e., where all injurers engage in $\hat{x}$ and that furthermore do not hold injurers liable at this activity level. In Lemma 1 we show that any such allocation can also be implemented by a negligence rule with the due activity level $\hat{x}$. Accordingly, if we want all injurers to engage in the same activity level and furthermore that injurers are not liable at this value to create proper incentives for victims, we need not look for whatsoever complicated liability rules; a negligence rule always does the job for us. This result works in favour of the negligence rule using the reasonable man standard. Yet, in Proposition 1 we show that allocations where all injurers engage in the same activity level can always be improved upon. We find that liability rules under which both injurers and victims bear a positive amount of the accident costs improve welfare relative to negligence rules. Finally, in Proposition 2 we fully characterize the optimal liability rule for the case where $H(\cdot)$ is additively separable in the activity levels. The proofs of Lemma 1 and Proposition 1 are relegated to the Appendix.

Lemma 1: *Consider any allocation* $(\hat{X}, \hat{Y}) = \big((\hat{x}, ..., \hat{x}), (\hat{y}_1, ..., \hat{y}_n)\big)$ *implemented by any liability rule* $l(\cdot)$ *with* $l(\hat{x}, \hat{y}_j) = 0$ *for all components* $\hat{y}_j$ *in* $\hat{Y}$. *Then* $(\hat{X}, \hat{Y})$ *can also be implemented by the negligence rule*

$$\hat{l}(x, y) = \begin{cases} 0, & \textit{if } x \leq \hat{x}; \\ H(x, y), & \textit{otherwise.} \end{cases}$$

The idea behind the proof is straightforward. A negligence rule incorporates the highest possible penalty for raising the activity level beyond $\hat{x}$; the liability switches from no to strict liability. Since injurers chose $\hat{x}$ under the old rule $l(\cdot)$ they will continue to do so under the negligence rule $\hat{l}(\cdot)$ that uses the most drastic penalty scheme. By introducing $\hat{l}(\cdot)$ we do not alter the injurers' behaviour nor their liability status at the value $\hat{x}$. Accordingly, victims stick to the choice of $\hat{Y}$.

Having established this prerequisite, we may now show that any allocation of the form $(\hat{X}, \hat{Y})$ implemented by a rule that holds injurers not liable at $\hat{x}$ can be improved upon.

Proposition 1: *Any allocation* $(\hat{X}, \hat{Y}) = \big((\hat{x}, ..., \hat{x}), (\hat{y}_1, ..., \hat{y}_n)\big)$ *implemented by any liability rule* $l(\cdot)$ *with* $l(\hat{x}, \hat{y}_j) = 0$ *for all components* $\hat{y}_j$ *of* $\hat{Y}$ *is not second-best optimal.*

The ideas behind this rather lengthy proof are as follows. From Lemma 1 we know that any allocation of the form $(\hat{X}, \hat{Y})$ can also be implemented by a negligence rule with a due activity level $\hat{x}$. It is thus sufficient to show that any such allocation implemented by the corresponding negligence rule can be improved upon.

An allocation $(\hat{X}, \hat{Y})$ can give rise to the following two scenarios: either *all* injurers engage in too low activity or *some* injurers engage in too high activity. Consider first the case where the value of $\hat{x}$ is so low that all injurers engage in a too low activity level, i.e., $U_1'(\hat{x}) \geq \sum_j H_x(\hat{x}, \hat{y}_j)$. If we succeed to implement a new allocation where all injurers increase their activity level by some small $\delta > 0$ while victims stick to their choice of $\hat{Y}$, we obviously increase welfare because $U_i(\cdot)$ is concave and $p(\cdot, y)$ convex. This task is fairly simple. We confront injurers with a liability rule that charges them a constant small amount over the interval $[\underline{x}, \hat{x} + \delta]$ and total damages otherwise. The small amount is just equal to the rise in expected damages due to the increase δ in the injurers' activity. Since $U_i'(\hat{x}) \geq \sum_j H_x(\hat{x}, \hat{y}_j)$, $i = 1, ..., n$, all injurers are happy to raise their activity level. Their utility increases by more than their expected payments. Given that all injurers pick $\hat{x} + \delta$, victims face the same payment schedule under the new scheme as they did under the negligence rule. Consequently, they stick to the choice of $\hat{Y}$.

It follows from this argument that any allocation $(\hat{X}, \hat{Y})$ where all injurers engage in too low activity cannot be optimal. Thus, only allocations where some injurers engage in too high activity, i.e., where $U_1'(\hat{x}) < \sum_j H_x(\hat{x}, \hat{y}_j)$, remain candidates for second-best optima.

In this case the implementation of a welfare increasing allocation turns out to be trickier. We exploit the following nice property of negligence rules. Since injurers are not liable at the due activity level, victims bear the entire accident costs and their choice of $\hat{Y}$ is efficient given $\hat{X}$ which means $\partial W(\hat{X}, \hat{Y})/\partial y_j = 0$, $j = 1, ..., n$. Accordingly, by the envelope theorem, any small change of the victims' activity levels has no first-order effect on welfare. All we have to do is to implement a welfare increasing allocation of the injurers' activity levels that results in a small change of the victims' choices.

Since injurer # 1 engages in too high activity, we can increase welfare by lowering his activity level by some small $\delta > 0$ while the other injurers stick to $\hat{x}$. This is accomplished by the following liability rule. Injurers are not liable over the interval $[\underline{x}, \hat{x} - \delta]$, they pay a small amount over the interval $(\hat{x} - \delta, \hat{x}]$, and are strictly liable otherwise. The small amount is constructed as follows. Injurer # 1, the marginal utility underdog, prefers $\hat{x} - \delta$ in order not to be liable while the other injurers are happier to stick to $\hat{x}$ and pay the (small) bill. Given the injurers' behaviour, victims no longer pay for the entire damages and increase their activity choices accordingly. Yet, the rule is constructed such that this increase is small and thus has no first-order effect on welfare. Consequently, a liability rule that makes injurers bear a positive amount of the accident costs improves welfare relative to a negligence rule using the reasonable man standard.

Notice that our sharing rule exhibits properties similar to comparative negligence rules. Under comparative negligence rules accident costs are divided between the two parties according to their relative fault.[8] Under our liability rule an agent's damage payment (weakly) increases with his own activity level as is the case under comparative negligence. Under our rule an injurer's damage payment is not monotonic in the victims' activity level. Under comparative negligence an injurer's *share* of the accident costs (weakly) decreases when victims raise their activity level. However, it is unclear whether the injurer's damage payment decreases under comparative negligence. If victims raise their activity level, an injurer's share of the loss goes down but at the same time the loss goes up, making the overall effect on the injurer's damage payment ambiguous.

Let us now fully characterize the optimal liability scheme in case that $H_{xy} = 0$. In this special case we find a rule that actually implements the first-best allocation (X^*, Y^*).

Proposition 2: *Suppose $H_{xy} = 0$ so that $H(x, y) = g(x) + f(y)$. Then the first-best allocation (X^*, Y^*) is implemented by the liability rule $l(x, y) = g(x)$.*

<u>Proof:</u> Under $l(\cdot)$ injurer $i, i = 1, ..., n$, maximizes

$$U_i(x) - ng(x)$$

with the first-order conditions

$$U_i'(x_i) = ng_x(x_i) \quad i = 1, ..., n, \tag{4}$$

while victim $j, j = 1, ..., n$, maximizes

$$V_j(y) - nf(y)$$

with the first-order conditions

$$V_j'(y_j) = nf_y(y_j) \quad j = 1, ..., n. \tag{5}$$

But (4) and (5) are the first-order conditions for the social optimum (X^*, Y^*) if $H_{xy} = 0$.

Q.E.D.

If $H(\cdot)$ is additively separable, we can impose the full marginal damage on injurers as well as victims and at the same time split up the accident costs between the two parties. Injurers and victims bear a positive fraction of the damage. The costs each party incurs under $l(\cdot)$ are increasing in the activity level.

Note that this particular case corresponds to the one analysed by Green (1976). Green restricts his attention to the class of liability rules that define due activity levels for injurers and/or victims. He does not derive the scheme $l(\cdot)$ that actually implements the first-best outcome. Note further that Green's statement that rules which allow for the sharing of costs cannot, by their very nature, impose the full marginal impact on each group and therefore are not first-best optimal is not generally true. In the additively separable case which Green considers, the sharing rule $l(\cdot)$ imposes the full marginal impact on each group and therefore implements the first-best allocation of the activity levels.[9]

Let us finally discuss the informational requirements of the different liability rules. Our rule requires that a planner knows the preferences of each type of agent and the damage technology. The planner need not know which particular individual is of what type. The reasonable man rule has the same informational requirements. To determine the reasonable man standard, i.e., the reasonably efficient due activity level, a planner needs to know the preferences of each type of agent and the harm function $H(\cdot)$. See Shavell (1987, p. 86-89). The incremental Learned Hand rule has the strongest informational requirements. A judge need not only know the preferences of each type of injurer; he must also ascertain which particular injurer is of what type. Moreover, the judge needs to know the preferences of each type of victim and the harm technology $H(\cdot)$ in order to compute Y^*. Then he can check whether $U_i'(x) < (\geq) \sum_j H_x(x, y_j^*)$, i.e., whether the injurer is strictly liable or not liable.

V. Conclusions

In this paper we have shown that negligence rules are not efficient in providing the correct incentives if individuals are non-identical. A rule that shares accident costs induces injurers to sort themselves in a welfare increasing way. Victims no longer bear the entire accident costs under the sharing rule and will therefore increase their activity choices compared to a negligence rule. Yet, since victims choose the efficient levels given the injurers' inefficient choice under a negligence rule, any small change in the victims' activity has no first-order effect on welfare. Note further that our results still hold if we allow victims to choose a second variable that captures, e.g., their care level.

We thus find that the recent movement in the U.S. towards sharing rules can be justified on efficiency grounds without relying on risk-sharing aspects.

Appendix A

Proof of Lemma 1: Under the negligence rule $\hat{l}(\cdot)$ we have that

$$U_i(\hat{x}) > U_i(x) - \sum_j \hat{l}(x, \hat{y}_j) = U_i(x) \quad \forall x < \hat{x} \quad \text{and}$$

$$U_i(\hat{x}) \geq U_i(x) - \sum_j l(x, \hat{y}_j) \geq U_i(x) - \sum_j \hat{l}(x, \hat{y}_j) \quad \forall x \geq \hat{x}, i = 1, ..., n,$$

because $l(x, \hat{y}_j) \leq \hat{l}(x, \hat{y}_j) = H(x, \hat{y}_j) \quad \forall x > \hat{x}$. Thus, given that victims engage in $\hat{Y}$, all injurers choose $\hat{x}$ under the negligence rule $\hat{l}(\cdot)$.

The vector $\hat{Y}$ is the optimal choice for victims under $l(\hat{x}, \cdot)$, i.e., $\hat{Y}$ satisfies

$$V_j(\hat{y}_j) - \sum_i H(\hat{x}, \hat{y}_j) \geq V_j(y) - \sum_i [H(\hat{x}, y) - l(\hat{x}, y)] \quad \forall y \in [\underline{y}, \bar{y}], j = 1, ..., n.$$

Since $l(\hat{x}, y) \geq 0 \ \forall y \in [\underline{y}, \bar{y}]$ we have that

$$V_j(\hat{y}_j) - \sum_i H(\hat{x}, \hat{y}_j) \geq V_j(y) - \sum_i H(\hat{x}, y) \quad \forall y \in [\underline{y}, \bar{y}], j = 1, ..., n. \tag{6}$$

Hence, victims continue to choose $\hat{Y}$ under $\hat{l}(\hat{x}, \cdot)$. Note that the conditions (6) further imply that victims engage in the efficient activity levels given the injurers' inefficient choice of $\hat{X}$, i.e., we have $\partial W(\hat{X}, \hat{Y})/\partial y_j = 0, \quad j = 1, ..., n$.

Q.E.D.

Proof of Proposition 1: Suppose not. By Lemma 1 we have that any allocation of the form $(\hat{X}, \hat{Y})$ can be implemented by a negligence rule. It is thus sufficient to show that any allocation of the activity levels $(\hat{X}, \hat{Y})$ implemented by the negligence rule

$$\hat{l}(x, y) = \begin{cases} 0, & \text{if } x \leq \hat{x}; \\ H(x, y), & \text{otherwise,} \end{cases}$$

can be improved upon.

There are two cases to consider, namely whether $U_1'(\hat{x}) \geq (<) \sum_j H_x(\hat{x}, \hat{y}_j)$. In both cases we will alter the injurers' activity choices by a new liability rule such that welfare increases.

Consider first the case where $U_1'(\hat{x}) \geq \sum_j H_x(\hat{x}, \hat{y}_j)$ which implies that each injurers' marginal utility does not exceed the marginal expected damage. By a new liability rule we will implement a new allocation where all injurers increase their activity slightly. This new allocation gives rise to higher welfare because $U_i'(\hat{x}) > \sum_j H_x(\hat{x}, \hat{y}_j) \quad i = 2, ..., n$. Thus, any allocation $(\hat{X}, \hat{Y})$ with $U_1'(\hat{x}) \geq \sum_j H_x(\hat{x}, \hat{y}_j)$ can be improved upon, contradicting the optimality of $(\hat{X}, \hat{Y})$.

Consider the new liability rule

$$l(x, y) = \begin{cases} H(\hat{x} + \delta, y) - H(\hat{x}, y), & \text{if } x \leq \hat{x} + \delta, \quad \delta > 0; \\ H(x, y), & \text{otherwise.} \end{cases}$$

Suppose injurers pick $\tilde{X} = (\hat{x} + \delta, ..., \hat{x} + \delta)$. Then we have

$$V_j(\hat{y}_j) - \sum_i [H(\hat{x} + \delta, \hat{y}_j) - l(\hat{x} + \delta, \hat{y}_j)] =$$

$$V_j(\hat{y}_j) - \sum_i H(\hat{x}, \hat{y}_j) \geq V_j(y) - \sum_i H(\hat{x}, y) \quad \forall y \in [\underline{y}, \bar{y}], j = 1, ..., n,$$

i.e., victims continue to choose $\hat{Y}$ under the new rule.

Let us now determine the injurers' optimal choice under $l(\cdot)$ given that victims engage in $\hat{Y}$. For $x \in [\underline{x}, \hat{x} + \delta]$ we obviously have that $\hat{x} + \delta$ is the optimal activity level. Because $U_i'(\hat{x}) \geq \sum_j H_x(\hat{x}, \hat{y}_j)$, $i = 1, ..., n$, for sufficiently small δ, $U_i(\hat{x} + \delta) - \sum_j H(\hat{x} + \delta, \hat{y}_j) \geq U_i(\hat{x}) - \sum_j H(\hat{x}, \hat{y}_j)$. Hence, for $(\hat{x} + \delta, \bar{x}]$ we have $U_i(\hat{x} + \delta) - \sum_j l(\hat{x} + \delta, \hat{y}_j) \geq U_i(\hat{x}) \geq U_i(x) - \sum_j H(x, \hat{y}_j)$ by the choice under the original rule. That is, all injurers are happy to engage in $\hat{x} + \delta$. The liability rule $l(\cdot)$ thus implements a new allocation that increases welfare relative to $(\hat{X}, \hat{Y})$, contradicting the optimality of $(\hat{X}, \hat{Y})$.

Consider now the case where $U_1'(\hat{x}) < \sum_j H_x(\hat{x}, \hat{y}_j)$ which means that injurer one's marginal utility is less than the expected marginal damage. We will now implement an allocation where injurer one lowers his activity level slightly. This new allocation gives rise to higher welfare because $U_1'(\hat{x}) < \sum_j H_x(\hat{x}, \hat{y}_j)$.

Consider the new liability rule

$$l(x,y) = \begin{cases} 0, & \text{if } x \leq \hat{x} - \delta, \quad \delta > 0; \\ \epsilon L(y), & \text{if } \hat{x} - \delta < x \leq \hat{x}, \quad \epsilon \geq 0; \\ H(x,y), & \text{otherwise,} \end{cases}$$

where

$$L(y) = \begin{cases} H(\hat{x}, y) - H(\hat{x}, \hat{y}_j - \lambda), & \text{if } y \in [\hat{y}_j \pm \lambda], \\ & 0 < 2\lambda < \min_{j=1,\dots,n-1}[\hat{y}_{j+1} - \hat{y}_j], j = 1, \dots, n; \\ H(\hat{x}, y), & \text{otherwise.} \end{cases}$$

Suppose that under $l(\cdot)$ victims engage in some $\tilde{Y}$ such that $0 \leq \tilde{y}_j - \hat{y}_j < \lambda, j = 1, \dots, n$. Injurer one then prefers $\hat{x} - \delta$ to $\hat{x}$ while for injurer $i, i = 2, \dots, n$ the opposite is true iff

$$\frac{U_i(\hat{x}) - U_i(\hat{x} - \delta)}{\sum_j [H(\hat{x}, \tilde{y}_j) - H(\hat{x}, \hat{y}_j - \lambda)]} \geq \epsilon \geq \frac{U_1(\hat{x}) - U_1(\hat{x} - \delta)}{\sum_j [H(\hat{x}, \tilde{y}_j) - H(\hat{x}, \hat{y}_j - \lambda)]}.$$

Since $U_1'(\cdot) < U_i'(\cdot), \quad i = 2, \dots, n, \forall\lambda > 0$ and $\forall\delta > 0$ there exists some $\epsilon > 0$ that satisfies the chain of inequalities. Note that ϵ approaches zero as δ goes to zero.

Next suppose that injurer one engages in $\hat{x} - \delta$ while injurer $i, i = 2, \dots, n$ engages in $\hat{x}$ so that the injurers' activity levels are given as $\tilde{X} = (\hat{x} - \delta, \hat{x}, \dots, \hat{x})$. Victim $j, j = 1, \dots, n$, then maximizes

$$V_j(y_j) - H(\hat{x} - \delta, y_j) - (n-1)[H(\hat{x}, y_j) - \epsilon L(y_j)].$$

Denote the maximizer by $\tilde{y}_j(\epsilon)$. Suppose $\epsilon = 0$. By making δ slightly positive, victim j increases his activity by

$$\frac{H_{xy}(\hat{x}, \hat{y}_j)}{nH_{yy}(\hat{x}, \hat{y}_j) - V''(\hat{y}_j)} \geq 0.$$

Now increase ϵ slightly at $\epsilon = 0$. Then victim j increases his activity level slightly (see Appendix B for a proof) by

$$\frac{dy_j}{d\epsilon} = \frac{(n-1)H_y(\hat{x}, \hat{y}_j)}{nH_{yy}(\hat{x}, \hat{y}_j) - V_j''(\hat{y}_j)} > 0.$$

The first-order effects on welfare of the victims' changes are zero by the envelope theorem.

Finally, we have to check that injurers choose $\tilde{X}$ under $l(\cdot)$ given that victims engage in $\tilde{Y}(\epsilon)$. By the construction of $l(\cdot)$, it is obvious that $\hat{x} - \delta$ is the globally optimal choice for injurer one. It is furthermore clear that $\hat{x}$ is optimal for injurer $i, i = 2, ..., n$, $\forall x \in [\underline{x}, \hat{x}]$. It remains to be shown that $\hat{x}$ is individually optimal for $x \in [\hat{x}, \bar{x}]$, i.e., $U_i(\hat{x}) - \sum_j \epsilon[H(\hat{x}, \tilde{y}_j(\epsilon)) - H(\hat{x}, \hat{y}_j - \lambda)] \geq U_i(x) - \sum_j H(x, \tilde{y}_j(\epsilon)) \quad \forall x \in [\hat{x}, \bar{x}], i = 2, ..., n$. Note that for $\epsilon = 0$ the incentive constraints are satisfied as $\tilde{Y}(0) \geq \hat{Y}$. If we make ϵ slightly positive, injurer i's net utility when choosing $\hat{x}$ decreases by $\sum_j [H(\hat{x}, \hat{y}_j) - H(\hat{x}, \hat{y}_j - \lambda)] > 0$. For $x > \hat{x}$ the net utility decreases by $\sum_j H_y(\hat{x}, \hat{y}_j) dy_j / d\epsilon > 0$. If we choose λ small enough, we have that $\sum_j [H(\hat{x}, \hat{y}_j) - H(\hat{x}, \hat{y}_j - \lambda)] < \sum_j H_y(\hat{x}, \hat{y}_j) dy_j / d\epsilon$. Accordingly, $\hat{x}$ is the globally optimal choice for injurer $i, i = 2, ..., n$.

Hence, the liability scheme $l(\cdot)$ implements the allocation $(\tilde{X}, \tilde{Y})$ that increases welfare relative to $(\hat{X}, \hat{Y})$, contradicting the optimality of $(\hat{X}, \hat{Y})$.

Q.E.D.

Appendix B

In this appendix we show that making ϵ slightly positive changes the victims' activity choice slightly.

Lemma 2: *Let $\phi(y)$ be a continiuous function on a compact set $[\underline{y}, \bar{y}]$ which achieves a unique maximum at y^*. If ϕ^n is a sequence of functions which approach ϕ uniformly and which achieve a maximum on $[\underline{y}, \bar{y}]$, then the set of maximizers of ϕ^n approach y^*.*

<u>Proof:</u> Suppose not. Then there exists a sequence $y^n \to \hat{y}, \hat{y} \neq y^*, y^n \in \text{argmax}\ \phi^n$. Continuity of ϕ and the fact that ϕ^n converges uniformly to ϕ imply that $\phi^n(y^n) \to \phi(\hat{y})$. But $\phi^n(y^*) \to \phi(y^*)$ and $\phi(y^*) > \phi(\hat{y})$. A contradiction.

Q.E.D.

Now let ϵ^n be any sequence going to zero. Let ϕ^n be the upper envelope (see Royden (1968 p. 49-50)) for

$$V(y) - H(\hat{x} - \delta, y) - (n-1)[H(\hat{x}, y) - \epsilon^n L(y)].$$

ϕ^n is upper semicontinuous and converges uniformly to

$$V(y) - H(\hat{x} - \delta, y) - (n-1)H(\hat{x}, y).$$

Hence, Lemma 2 applies.

Q.E.D.

Footnotes

1) The assumption that the number of victims equals the number of injurers is made for notational convenience.

2) We have formalized the externality problem in terms of activity levels. Parts of the literature (see, e.g., Shavell (1980)) deal with the case where x and y denote care levels. The injurers' utility $a(x)$, the victims' utility $b(y)$, and the expected harm $t(x, y)$ decrease with care. In this case let $U(x) \equiv a(\bar{x} - x + \underline{x})$, $V(y) \equiv b(\bar{y} - y + \underline{y})$, and $H(x,y) \equiv t(\bar{x} - x + \underline{x}, \bar{y} - y + \underline{y})$ and work with $U(\cdot)$, $V(\cdot)$, and $H(\cdot)$ instead of $a(\cdot)$, $b(\cdot)$, and $t(\cdot)$. Accordingly, an increase in care corresponds to a reduction in the activity level, the lowest care level corresponds to the highest activity level etc. Thus, by the above transformation all of our results immediately apply to models set up in terms of care instead of activity.

3) An interior maximizer of $W(\cdot)$ follows from the following assumptions. Let $nH_x(\underline{x}, \bar{y}) < U_1'(\underline{x}), nH_x(\bar{x}, \underline{y}) > U_n'(\bar{x}), nH_y(\bar{x}, \underline{y}) < V_1'(\underline{y})$ and $nH_y(\underline{x}, \bar{y}) > V_n'(\bar{y})$. These conditions imply that each individual's marginal utility at the lowest activity level exceeds the expected marginal damage and vice versa for the highest activity level, independent of what the other agents do. Furthermore, let $U_i(\underline{x}) > nH(\underline{x}, \bar{y})$ and $V_j(\underline{y}) > nH(\bar{x}, \underline{y}) \quad i, j = 1, ..., n$. Each individual's utility at the lowest activity level is higher than the expected damage, whatever the other agents do. Therefore, it is socially desirable that all individuals engage in their respective activity at a positive level.

4) Other examples abound: The probability and/or the degree of oil spills may depend on the speed at which ships use a sea lane. How carefully ice is cleared from sidewalks and the amount of foot traffic determine the number of broken legs. The number of hunting accidents depends on the amount of hunting and hiking in a forest.

5) We assume that disputes can be resolved without cost. See P'ng (1987) for a model with costly litigation. Moreover, agents correctly assess expected damages. See Endres (1989) for an analysis where agents misperceive the expected harm.

6) If $\hat{x}$ is not chosen high enough, then some injurer may pick $x > \hat{x}$. He is strictly liable

and thus distorts the victims' incentives because they no longer bear the entire accident costs. In what follows we will not deal with this uninteresting case.

7) Rubinfeld (1987) similarly to us argues that a negligence rule can be improved upon by a sharing rule. If a sharing rule is appropriately designed, it induces injurers with high marginal utility to engage in higher activity than injurers with low marginal utility which increases welfare. Yet, Rubinfeld does not take into account the victims' activity levels. If they are fixed, the rule of strict liability $l(x, y) = H(x, y)$ implements the first-best optimum and there is no need for second-best solutions. In case that victims adjust their choices, under a sharing rule they engage in higher activity than under a negligence rule because they bear only a part of the accident costs. This change in the victims' choices decreases welfare, making the overall effect on welfare ambiguous.

8) See Cooter and Ulen (1986) for a description of the various comparative negligence rules applied in the U.S.

9) See Emons and Sobel (1988) for more on first-best implementation with liability rules that allow for punitive damages.

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