On Capacity Precommitment Under Cost Uncertainty *

Armin Hartmann †

April 18, 2006

Abstract

This paper considers a capacity pre-commitment game where cost are unknown. We expand the model of Kreps and Scheinkman to the uncertainty case. Firms choose capacities under uncertainty about marginal costs in the price setting stage. The setup is plausible as in reality, firms do not know marginal costs exactly at the capacity stage. We can show that equilibria are necessarily asymmetric. While the weakly larger firm always behaves like a Cournot player, the opponent has an incentive to lower capacities. In an extension we show that if units costs differ, results change substantially.

Keywords: Bertrand-Edgeworth Competition, Capacity Pre-commitment, Cost Uncertainty
JEL: D21, D43, D44

---

*I am grateful for comments from Winand Emons, Philipp Ackermann, Alain Egli, Roland Hodler, Simon Loertscher, Andreas Niedermayer.

†University of Bern, Department of Economic Theory, Schanzeneckstrasse 1, P.O. Box 8573, CH-3001 Bern, Switzerland; armin.hartmann@vwi.unibe.ch

1 Introduction

The nature of short run competition is a great deal of discussion since almost two centuries. Within the last 100 years economists were basically divided between those who thought that firms set quantities and those who supposed that firms set prices.

On one side there were economists who supported the idea of Cournot, who proclaimed competition in quantities. Each oligopolist informs the auctioneer about the quantity he is willing to produce. The auctioneer computes the market clearing price and each firm is forced to produce exactly the named quantity. No additional trade is possible. Cournot-followers admired especially the valuable advantage that firms make positive profits in equilibrium. Other properties like the amazing computation and the fact that aggregate profits typically decrease in the number of firms patronized the leading position of Cournot.

On the other hand some economists fought for Bertrand and the notion that firms set prices and the auctioneer does not exist - or is at least not more than a mysterious approximation of a more complex price building process. With price setting firms the auctioneer is not needed anymore. It is the market that allocates market demand to the firm with the lowest price.

For decades it was not clear to which ideas economists gave priority. Mas-Colell et. al. (1988) aptly state that the Cournot model "gives the right answer for the wrong reason". None of the two models finally represented the reality. Both had drawbacks that contradicted real life observations in a fundamental way.

In 1983 David M. Kreps and José Scheinkman published their article "Quantity Precommitment and Bertrand Competition Yield Cournot Outcome" in the Bell Journal of Economics. They constructed a two stage model where in a first stage firms have to build up capacities and set prices afterwards. It turns out that in the unique subgame perfect equilibrium firms build up Cournot capacities and set capacity clearing prices in the second stage.

This model is appealing for several reasons. First, it satisfies the Bertrand critique. Firms set prices not quantities, at least in the short run. This implies that the market replaces the auctioneer. Second, the result seems to correspond to real life. Firms earn positive profits in equilibrium. Kreps and Scheinkman successfully connected the convincing story of Bertrand with the universally accepted result of Cournot. The paper finally yielded a new paradigm. In the long run firms compete in quantities, in the short run firms compete in prices. This new paradigm had a great influence in the theory of Industrial Organization. A lot of papers were published in the late 80's.

However, the model has different simplifications and drawbacks. Kreps and Scheinkman assume the efficient rationing rule. As capacities are finite, there may be cases where demand at a given price is bigger than the quantity of the firms (or one firm at least). The efficient rationing rule implies that every unit of the good is allocated to the consumer with the highest valuation for it. Several papers tried to analyze differences between efficient rationing, proportional rationing or even random rationing. It is true that the Cournot result is valid only for the efficient rationing rule. It is even correct that for other rationing rules firms do not necessarily play pure strategies. But we can state that the installed capacities are not very sensitive to changes in the rationing rule. Quantities are "almost" Cournot - whatever that means. But the key factor is a different one. We can verify the efficient rationing rule by economic introspection. Only the efficient rationing rule exhausts gains from trade. This assures that the allocation is stable. Finally, the formulation of the profit function is much easier for the case of efficient rationing. All these facts yielded this rationing rule to be the one commonly used in oligopoly models.

But the Kreps-Scheinkman paper faced additional critique. A puristic argument bases upon the fact that in the subgame starting at the second stage, firms have to compute a mixed strategy equilibrium. "People do not play mixed strategies", these people argue. There are different arguments to sustain the idea of mixed, especially fully mixed, strategies. We only want to stress an empirical point. There are several markets where the so called Edgeworth-cycles occur. Prices vary over time quite regularly on a special set. They never exceed a maximum and do not fall below a certain barrier. The most important example is the electricity market. A lot of empirical papers analyze these cycles in electricity markets. But there is also a huge list of papers that try to implement this outcome in a theoretical model, e.g. Maskin and Tirole (1988) or Burdett and Judd (1983). These dynamic oligopoly models have been tested empirically, e.g. the retail gasoline market by Michael Noel (2004).

Somewhat more basic simplifications in the Kreps-Scheinkman model are the perfect information and the equal cost assumption. It is not crazy to say that reality is much more complex. The equal cost assumption seems to be natural, the supporting argument a standard micro one. Suppose there are two firms with zero fixed costs and unequal marginal costs. In this case the inefficient firm should drop out of the market. But if we deal with capacity constraints it would be incorrect to assume that only the efficient technology should survive. There are two effects: By building up higher capacities the firm with the lower costs can price the other firm out of the market. But there is an effect that goes into the other direction. Higher capacities imply a lower markup. The lower the aggregate capacities, the higher the price in general. That is why the efficient firm has an incentive to build up low capacities and to sell at a high price. Therefore, the inefficient firm will have a residual demand. It will set a supracompetitive price on this residual demand will make positive profits. The inefficient technology does not vanish necessarily. As far as we know there is not much empirical evidence that inefficient firms rather survive in markets with capacity constraints. But we would dynamic inefficiency expect to be higher in markets with capacity constraints.

Nevertheless, there are also arguments to sustain the equal cost assumption.

As capacities are flexible in the long run, the efficient firm should increase capacities. On the other hand the inefficient firm always has an incentive to decrease marginal costs. As high costs are easier to decrease than low costs, the difference between marginal costs should not be too large in the long run.

We can summarize that there are arguments for equal as well as for unequal costs. In line with this conclusion we assume in the first part equal costs, but we will relax this assumption in the extension part.

More important is the fact that different unit costs alter the outcome in the static Bertrand Edgeworth model. The appropriate source for the unequal-cost setup is an article by Dan Kovenock and Ray Deneckere. They analyzed the Bertrand Edgeworth game if unit costs differ. One of their main results is that in the two stage capacity pre-commitment case firms do not necessarily build up Cournot capacities. If unit costs difference is large enough, the more efficient firm has an incentive to price the other firm out of the market. Formal aspects in the second stage price setting game are awful. Non-connected equilibrium-strategy spaces, masspoints and apparently decreasing cumulative distribution functions occur during the computation process. Beside these unaesthetic problems their contribution is appealing. Since at least one firms earns the minmax profit, equilibrium profits are often quite easy to derive.

In this paper we try to emphasize the other important simplification in the Kreps-Scheinkman model. We expand the capacity pre-commitment case to cost uncertainty. Cost uncertainty is a crucial point for short run competition. We know that cost uncertainty induces positive expected profits in Bertrand equilibria (see Spulber 1995). We also know that cost uncertainty alters equilibria in the Cournot case (Basar 1978). We think it is natural to assume that firms know their costs in the short run - but in the long run they only have a belief about it. Consequently, we assume that firms have to build up capacities knowing only the distribution of the costs. In the second stage marginal costs realize and firms face full information Bertrand Edgeworth competition.

The discussion of cost uncertainty is new. In the literature we find papers about capacity pre-commitment under demand uncertainty. An important paper is by Reynolds and Wilson (2000). They show that equilibria may be asymmetric. This property jumps over to the cost uncertainty case. The argument is very similar. But our methodology differs fundamentally.

Both, demand and cost uncertainty are relevant issues for oligopoly theory. But we think that the cost uncertainty assumption is more realistic. We agree that in the long run cost and demand are uncertain. If capacity is installed, uncertainty about marginal costs should be smaller, or even vanish. This is not the case for demand uncertainty. If capacity is installed, there is no reason why demand should be full information now. On this account the story of cost uncertainty is superior.

We want to analyze which capacities firms build up and compare these to the Cournot benchmark. We can show that equilibria are necessarily asymmetric. The reaction function of the weakly larger firm is equal to a Cournot firm that has the lowest possible costs - independent of the cost distribution. If a symmetric equilibrium would exist, it would be the Cournot equilibrium with firms that have the lowest possible cost. But we show that this cannot be an equilibrium. Since at this equilibrium candidate marginal revenue of an additional unit of capacity is negative for the smaller firm this cannot be an equilibrium. So, in equilibrium the large firm chooses a capacity that is higher than the stated Cournot quantity and the small firm chooses a capacity that is lower. Equilibria in the price setting game necessarily occur in mixed strategies.

We do not want to discuss the case where firms know their costs before the capacity stage (private information case). This setup would imply that firms have the chance to signal the marginal costs to the opponent. But it would be impossible to find a unique perfect Bayes-Nash equilibrium. As in signalling games we can show almost everything, this question is of less interest for us. But this may nevertheless be a topic for additional research.

The paper is organized as follows. We first analyze the simple monopoly case. Section 3 introduces the model. Section 4 deals with the preliminaries of the full information Bertrand Edgeworth model. Section 5 solves the capacity precommitment under uncertainty model. Section 6 extends the model to different unit costs. Section 7 concludes the article.

2 The Simple Monopoly Case

To give an intuition of what follows we start with a very simple example. Suppose a monopoly firm faces the following two stage problem. At the first stage the firm has to install capacities k at costs $b(k)$. At the second stage the same monopoly firm will have to set prices. The firm knows second stage demand $D(p)$ at the first stage. The monopolist faces cost uncertainty. At the first stage the monopoly firm knows that in the second stage it will have marginal costs drawn from a distribution $F(c)$ with support $(\underline{c}, \overline{c})$. In the second stage the firm can produce at costs c up to capacity k. Production beyond k is infinitely costly. We can represent the timing of the game as follows:

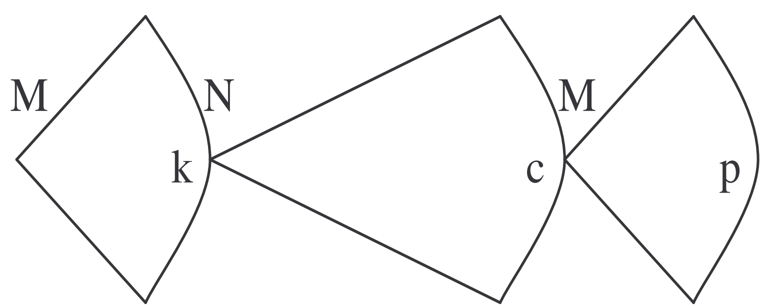

Figure 1: The Timing in the Monopoly Case

In this case we surely have to assume positive costs to install capacity $b(k) > 0$. Since there is no competition effect that could firm force to set low capacities a monopoly would set infinite capacity whenever capacity is free.

We can solve this simple decision problem without any strategic interaction by backward induction. We first have a look at stage two. As we know that the monopoly price is a nondecreasing function of marginal costs, higher costs will in general imply higher prices. For a given k the capacity constraint will be binding if and only if costs are low enough. If a firm has bad luck, it may draw such high costs that the installed capacity may be higher than the effective demand at the chosen price. We assume that this excess capacity does not generate any costs (capacity can be burned costlessly). The profit of a monopolist for a chosen k and a realized cost c is

$$\Pi = (p^* - c)\min\left(D(p^*), k\right) - b(k)$$

The expected profit of the monopolist is

$$E\Pi = \int_{\underline{c}}^{\overline{c}} (p^* - c)\min\left(D(p^*), k\right) dF(c) - b(k)$$

To derive an explicit solution we assume that $D(p) = 1 - p$ and costs are uniformly distributed between 0 and 1. We assume convex capacity costs $b(k) = k^2$.

Obviously the monopolist will never set a price below the capacity clearing price $p = 1 - k$. At any price below, the monopolist can raise the price without decreasing demand. If at any chosen price the capacity constraint is binding, the monopolist should have chosen the capacity clearing price $p = 1 - k$. In this case the profit is equal to $(1 - k - c)k$. If the constraint is not binding the monopolist should have chosen the monopoly price $p = \frac{1+c}{2}$ and the profit will be $\Pi = \left(\frac{1-c}{2}\right)^2$. But the monopolist will increase the price beyond the capacity clearing price if and only if it increases the profit at this point i.e. if and only if $\frac{\partial \Pi}{\partial p}|_{p=1-k} > 0$. This is the case whenever $1 - 2(1-k) + c > 0$ or equivalently if $c > 1 - 2k$. Hence, we can rewrite the problem as

$$E\Pi = \int_0^{1-2k} (1-k-c)kdc + \int_{1-2k}^{1} \left(\frac{1-c}{2}\right)^2 dc - k^2$$

As $k < 1$ this is a concave function with the unique maximizer $k = 1 - \frac{\sqrt{3}}{2}$. The monopolist will set the price 0.866 if $c < 0.732$ and $\frac{1+c}{2}$ if $c \geq 0.732$. The expected profit is equal to 0.0327. The probability that the monopolist will not sell the whole capacity is $Pr(1-p < k) = 1 - 2k = 0.268$. This means that if nature draws costs that are higher than 0.732, the monopolist will not sell the whole capacity.

We have to point out that the expected profit under uncertainty is obviously lower than the profit of an expected-cost type under full information. For our example an expected cost type ($c = 0.5$) would set a capacity of $k = \frac{1}{8}$ and would have a profit of 0.047 for sure. The reason why an expected-cost type under full information sets a lower capacity and makes a higher profit is simple. The profit function is convex in the cost-type variable c. This implies that if costs are low, a firm will profit much from a high capacity. Even more than the loss if costs are high and not the whole capacity can be sold.

The monopoly example in its simplicity gives us an intuition of the quantity effect. A higher capacity yields higher profit whenever costs are not too high.

We now turn to the duopoly. Here we have the same quantity effect. But additionally, we will have a competition effect. Whenever at least one firm leaves positive residual demand, both firms will make positive profits. This creates an incentive to choose small capacities. With small capacities firms can set high prices on the residual demand.

We first explain the model. For the solution of the price setting game we rely on the models by Kreps-Scheinkman and Konvenock-Deneckere.

3 The Model

Two firms $i = 1, 2$ produce a homogenous good for which market demand is $D(p)$. We assume $D(0) > 0$, $D'(p) < 0$ and $\exists\ \hat{p}$ s.t. $D(\hat{p}) = 0$. In order to avoid second order conditions we assume that $2D'(p) + (p - c)D''(p) < 0\ \forall\ 0 < p < \hat{p}$. The competitive situation can be represented in a three-stage model. At the first stage, firms have to build up capacities k_i at (possibly zero) costs $b(k)$. Both firms have the same capacity costs. At the first stage they know market demand $D(p)$ and the cumulative density function of their own and the opponent's marginal costs $F_i(c_i) = F_j(c_j) = F(c)$ in the second stage. For simplicity we assume in this section that both firms will have the same marginal cost $c_i = c_j = c$ in the price setting stage. We will relax this later. We assume that the cost distribution is atomless and connected on the compact set $\mathcal{C} = \{c | c \in (0, \hat{p})\}$[1]. Fixed costs are zero for both firms. There are no restrictions on k_i .

At the second stage nature independently draws unit costs c from the distribution $F(c)$. Both firms observe the cost parameter c. With a capacity k_i, firm i can produce up to a quantity k_i at costs c. Production beyond k_i is infinitely costly.

In the final stage both firms simultaneously and independently have to set their prices. The market allocates demand to the firm with the lowest price up to its capacity. Firms can dispose excess capacity without cost.

The third stage game $(k_1, k_2, c, D(p))$ is now a standard full information Bertrand-Edgeworth game. The timing of this game is indicated in figure 2.

Since capacity of the firm with the lower price may be smaller than demand at this price we have to declare a rationing rule. Consistent with the main literature we make the assumption that demand is allocated efficiently. The consumers with the highest valuation for the good are served by the firm with the lower price. In

[1] We bound the distribution only for convenience. For $c > \hat{p}$ a Bertrand argument would apply.

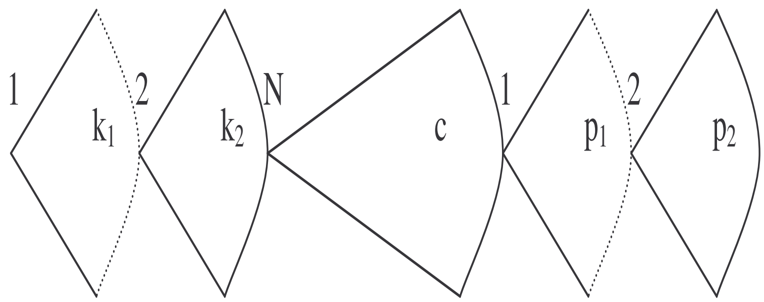

Figure 2: Timing

other words the overbidder i faces residual demand $D(p) - k_j$. We discussed pros and cons of this particular rationing rule in the introduction. While some papers showed that results crucially depend on this assumption it is still accepted. The most important reason is that this particular rationing rule exhausts gains from trade. There is no possibility that a consumer with low valuation can resell the product he bought for a low price to a consumer with high valuation.

With the efficient rationing rule expected profit of each firm can be written as:

$$\begin{aligned} E\pi_i\left(p_i\left(c_i\right), c_i\right) &= \left(p_i - c_i\right) \min\left(D_i\left(p_i\right), k_i\right) \Pr\left(p_i < p_j\right) \\ &+ \left(p_i - c_i\right) \min\left(\frac{D_i\left(p_i\right)}{2}, k_i\right) \Pr\left(p_i = p_j\right) \\ &+ \left(p_i - c_i\right) \min\left(\max\left(D\left(p_i\right) - k_j, 0\right), k_i\right) \Pr\left(p_i > p_j\right) \end{aligned}$$

The appropriate source for the price setting game is the Kreps-Scheinkman model. As unit costs are equal every result for their price setting game applies to our third stage.

We have to introduce some additional notation. We define $r_c(k_j)$ to be the Cournot reaction function of a firm i that has costs c. This function solves the problem:

$$\max_{k_i} \Pi_i = k_i(P(k_i + k_j) - c)$$

In the next section we discuss the main results of full information Bertrand Edgeworth competition.

4 Preliminaries: Bertrand Edgeworth Competition

There is a large literature on full information Bertrand-Edgeworth competition. This formulation connects the most important results from the long history starting with Beckmann (1967). The most results are by Kreps-Scheinkman for equal costs and Kovenock-Deneckere for differing unit costs. We adapt the results as much as needed for our purposes. We do not go into the details, especially not for the derivation of the equilibrium distributions for the fully mixed strategies.

The key theorem to derive equilibrium profits is theorem 1 by Kovenock and Deneckere.

Lemma 1 (Kovenock Deneckere 1996): At least one firm earns the minmax profit.

Proof of Lemma 1 : See Konvenock Deneckere lemma 1.

Lemma 1 implies that if firm i earns the minmax profit it simply maximizes the expression $(p_i - c_i)\min(\max(D(p_i) - k_j, 0), k_i)$. Intuitively the result is clear. Suppose the equilibrium occurs in mixed strategies where at least one firm does not have a masspoint at the supremum. This implies that one firm playing the supremum of its equilibrium distribution is undersold with probability 1. This is by definition the minmax profit. If both players had an equilibrium distribution with masspoints at the supremum it would be profitable to set all mass below this price (Bertrand argument).

This lemma simplifies the equilibrium computation especially in the fully mixed strategy region. But the equilibrium does not necessarily occur in mixed strategies.

Lemma 2 (Kovenock Deneckere 1996): Equilibria occur in pure strategies if

1. Both capacities are smaller than the Cournot reaction quantities. In this case both firms set the capacity clearing price $P(k_i + k_j)$.
2. $\min(k_i, k_j) > D(c)$. In this region both firms set $p = c$. Profits are zero.

Proof of Lemma 2 :

(1) Assume that capacities are smaller than the Cournot capacities. In this case the profit function of the overbidder $(p_i - c_i) \min\left(\max\left(D\left(p_i\right) - k_j, 0\right), k_i\right)$ is maximized by $P(k_i+k_j)$. This implies that the minmax profit is $H^* = k_i(P(k_i + k_j) - c)$. This completes the proof for equal costs.

If unit costs differ we have to add two lines. Analogously to Kovenock-Deneckere we define the infimum of the equilibrium price distribution as $\underline{p}$. For differing unit costs we know that $\underline{p_i} = \underline{p_j} = \underline{p}$. If a firm could earn more than the stated minmax profit it has to set a higher price with positive probability. But as $\Pi(p > \underline{p}) < H^*$ for the overbider we have a contradiction.

(2) Suppose $\min(k_i, k_j) > D(c)$. This implies that, independent of which firm sets the lower price, there is no residual demand for the other firm. With other words, the firms problem simplifies to the standard Bertrand problem. We restrict the strategy space to prices that do not lie below marginal costs and not above $\hat{p}$ in order to get a compact strategy space. For equal costs there is a unique equilibrium with $p = c$.[2] ■

We can see the regions graph 3:

Lemma 3 (Kovenock Deneckere 1996): Which firm earns the minmax profit is uniquely determined by the lowest price a firm can set in order to earn its

[2] If we do not have compact strategy spaces we typically have a continuum of equilibria.

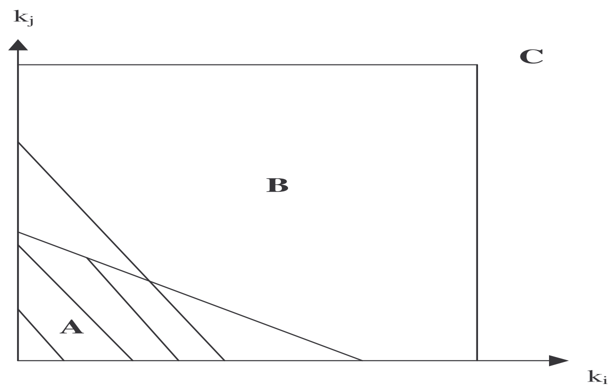

Figure 3: Strategy Regions in Full Information Bertrand Edgeworth

minmax profit. We call this price $\underline{p}$. For equal costs it follows directly that the firm with the larger capacity earns the minmax profit. If capacities are equal, both firms earn the minmax profit. The smaller firm, i.e. j, earns the profit of a firm that undersells the other by a price $\underline{p}_j$. This profit is $(\underline{p}_j - c)\min(D(\underline{p}_j), k_j)$. Following Kovenock-Deneckere we write this profit as $L(\underline{p})$.

Proof of Lemma 3 : See Konvenock Deneckere theorem 3 or Kreps-Scheinkman lemma 5.

In the capacity precommitment game of Kreps and Scheinkman firms build up Cournot capacities. If unit costs differ there is an additional feature. The low cost firm has a higher incentive to drive the other firm out of the market. If it favors this over selling to residual demand we are in the usual Bertrand region with differing costs. The low cost firm sets either the monopoly price or the marginal cost of the opponent. The other firm j plays any (mixed) strategy that deters firm i from raising its price.

5 Capacity Precommitment Under Uncertainty

First we have to analyze how the expected profit of a firm at stage one depends on its costs in stage 3. We know from the Bertrand-Edgeworth model that if capacities are low, firms play pure strategies in equilibrium. More concrete: If $k_i < r_c(k_j)$ and $k_j < r_c(k_i)$ both firms set prices equal to $P(k_i + k_j)$ and make profits $(P(k_i + k_j) - c)\, k_i$ for $i = 1, 2$.

Assume w.l.o.g. that $k_i \geq k_j$. This implies that the conditions that both firms lie in this region are uniquely determined by the condition $k_i \leq r_c(k_j)$ because this condition implies the one for the weakly smaller firm.

The reaction function fulfills the property $P(k_i + k_j) - c + k_i P'(k_i + k_j) = 0$. We solve for the costs $c = P(k_i + k_j) + k_i P'(k_i + k_j)$. The condition $k_i \leq r_c(k_j)$ is fulfilled whenever $c \leq P(k_i + k_j) + k_i P'(k_i + k_j)$. So, if costs are smaller for both firms, firms play pure strategies $p = P(k_i + k_j)$. The first part of the equilibrium pricing strategy is

$$p = P(k_i + k_j) \text{ if } \underline{c} \leq c \leq P(k_i + k_j) + k_i P'(k_i + k_j)$$

As 0 is always part of the equilibrium distribution this region is not empty.

If capacity is high, it may be that the capacity constraint is not binding. If $\min(k_i, k_j) > D(c)$ we are in the standard Bertrand region where the unique Nash Equilibrium implies that both firms make zero profits. We solve for c and get because of monotonicity the barrier $c = D^{-1}(\min(k_i, k_j)$.

If costs are high, firms have to set high prices. Setting high prices, firms will sell small quantities and the "probability" that capacities do not matter is much higher. This implies that the condition $\min(k_i, k_j) > D(c)$ is fulfilled whenever $c > D^{-1}(\min(k_i, k_j)$. In short, costs higher than $D^{-1}(\min(k_i, k_j)$ imply pure Bertrand competition. Therefore the second part of the equilibrium pricing strategy is

$$p = c \text{ if } D^{-1}(\min(k_i, k_j) \leq c \leq \hat{p}$$

Again our assumptions assure that his region is not empty.

When costs lie in between those two cases firms play fully mixed strategies $\phi_i(p)$ according to lemma 6 of Kreps-Scheinkman or lemma 2 ff of Kovenock-Deneckere. From Kreps and Scheinkman we know that the weakly larger firm earns the minmax profit. We write this profit as $r(k_j)P(r(k_j) + k_j) - c)$. The weakly smaller firms j earns the profit $(\underline{p}_j - c)\min(D(\underline{p}_j), k_j)$ with $\underline{p}$ implicitly defined by $H_j^* = (\underline{p} - c)k_i$. We write this profit shortly as $L\left(\frac{H^*}{k_i} + c\right)$.

We can summarize these consideration about the cost regions in graph 4:

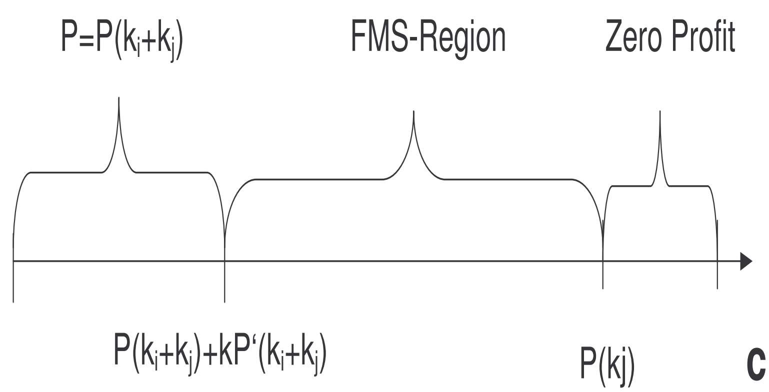

Figure 4: Cost Regions

We can now rewrite the expected profit of the weakly larger firm as :

$$\begin{aligned}
E\Pi_i &= \int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} k_i\left(P(k_i+k_j) - c\right) dF(c) \\
&+ \int_{P(k_i+k_j)+k_iP'(k_i+k_j)}^{P(k_j)} r_c(k_j)\left(P(r(k_j)+k_j) - c\right) dF(c) \\
&+ \int_{P(k_j)}^{\hat{p}} 0 dF(c)
\end{aligned}$$

As equilibria are not necessarily symmetric, we also have to compute the expected profit function of the weakly smaller firm. For the low and the high cost region the derivation is equal. In the mixed strategy region expressions will differ. By Kovenock-Deneckere, the firm that does not get the minmax profit, e.g. firm j, receives a payoff $L(\underline{p})$. This $\underline{p}$ can be computed as $\frac{H_i}{k_i+c}$. As the minmax profit H_i is computed easily we can write the problem of the weakly smaller firm as:

$$
\begin{aligned}
E\Pi_i \quad &= \int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} k_j \left(P(k_i+k_j)-c\right) dF(c) \\
&+ \int_{P(k_i+k_j)+k_iP'(k_i+k_j)}^{P(k_j)} L\left(\frac{H^*}{k_i}+c\right) dF(c) \\
&+ \int_{P(k_j)}^{\hat{p}} 0 dF(c)
\end{aligned}
$$

With this expressions it is now easy to state the theorem.

Theorem 1 : The equilibria in the capacity pre-commitment game are necessarily asymmetric. The reaction function of the weakly larger firm necessarily coincides with the Cournot reaction function of a firm with the lowest possible cost. The asymmetric equilibria have to be computed through the intersection of the reaction functions.

In the third stage firms play

$$
p\left(k_i,k_j,c,D(p)\right) = \begin{cases} \phi(p) & \text{if} \quad 0 < c < D^{-1}\left(\min(k_i,k_j)\right) \\ c & \text{if} \quad D^{-1}(\min(k_i,k_j)) \leq c \leq \overline{c} \end{cases}
$$

with $\phi(p)$ according to Kreps-Scheinkman.

Proof of Theorem 1 :

By lemma 3 we know that the expected payoff of the weakly larger firm i is

$$\begin{aligned} E\pi &= \int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} k_i\left(P\left(k_i+k_j\right)-c\right)dF\left(c\right) \\ &+ \int_{P(k_i+k_j)+k_iP'(k_i+k_j)}^{P(k_j)} r_c\left(k_j\right)\left(P\left(r_c\left(k_j\right)+k_j\right)-c\right)dF\left(c\right) \end{aligned}$$

By maximizing over k_i we find the reaction function of firm i $r_c(k_j)$.

$$\begin{aligned} \frac{\partial E\pi}{\partial k_i} &= \int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} \left(P\left(k_i+k_j\right)-c\right)+k_iP'\left(k_i+k_j\right)dF\left(c\right) \\ &+\left[k_i\left(P\left(k_i+k_j\right)-\left(P\left(k_i+k_j\right)+k_iP'\left(k_i+k_j\right)\right)\right)\right] \\ &*\left[2P'\left(k_i+k_j\right)+k_iP''\left(k_i+k_j\right)\right] \\ &-\left[\begin{array}{c} r\left(k_j,P\left(k_i+k_j\right)+k_iP'\left(k_i+k_j\right)\right) \\ *\left(P\left(r\left(k_j,P\left(k_i+k_j\right)+k_iP'\left(k_i+k_j\right)\right)+k_i\right)-P\left(k_i+k_j\right)-k_iP'\left(k_i+k_j\right)\right) \end{array}\right] \\ &*\left[2P'\left(k_i+k_j\right)+k_iP''\left(k_i+k_j\right)\right]=0 \end{aligned}$$

$$\begin{aligned} &\int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} \left(P\left(k_i+k_j\right)-c\right)+k_iP'\left(k_i+k_j\right)dF\left(c\right) \\ &+k_iP\left(k_i+k_j\right)-P\left(k_i+k_j\right)-k_iP'\left(k_i+k_j\right) \\ &-r\left(k_j\right)\left(P\left(r\left(k_j\right)+k_i\right)+P\left(k_i+k_j\right)+k_iP'\left(k_i+k_j\right)\right)=0 \end{aligned}$$

$$\begin{aligned} &\int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} \frac{\left(P\left(k_i+k_j\right)-c\right)+k_iP'\left(k_i+k_j\right)}{\left[2P'\left(k_i+k_j\right)+k_iP''\left(k_i+k_j\right)\right]}dF\left(c\right) \\ &+k_i\left(P\left(k_i+k_j\right)-P\left(k_i+k_j\right)-k_iP'\left(k_i+k_j\right)\right) \\ &-r\left(k_j\right)|_{c=P(k_i+k_j)+k_iP'(k_i+k_j)}\left(\begin{array}{c} P\left(r\left(k_i\right)|_{c=P(k_i+k_j)+k_iP'(k_i+k_j)}+k_i\right) \\ +P\left(k_i+k_j\right)+k_iP'\left(k_i+k_j\right) \end{array}\right)=0 \end{aligned}$$

$$\begin{aligned} &\int_0^{P(k_i+k_j)+k_iP'(k_i+k_j)} \frac{\left(P\left(k_i+k_j\right)-c\right)+k_iP'\left(k_i+k_j\right)}{\left[2P'\left(k_i+k_j\right)+k_iP''\left(k_i+k_j\right)\right]}dF\left(c\right) \\ &+\left(k_i-r_0\left(k_j\right)\right)*\left(P\left(k_i+k_j\right)-P\left(k_i+k_j\right)-k_iP'\left(k_i+k_j\right)\right)=0 \end{aligned}$$

$$\text{if } k_i = r_0\left(k_j\right) \rightarrow P\left(k_i + k_j\right) + k_i P'\left(k_i + k_j\right) = 0$$

This expression is equal to the Cournot reaction function of a firm that has costs 0. This implies that the unique symmetric equilibrium, if it exists, is the Cournot equilibrium with ($c = 0$). The bad news is that such an equilibrium does not exist.

We do the same thing for the weakly smaller firm. Now things are a little bit more complicated but the procedure is the same. The expression for the reaction function cannot be written in a compact form as for the weakly larger firm.

But if we check the equilibrium candidate $k_i = k_j = r_0$ we see that the first order condition for firm 2 is not satisfied. This completes the proof. ■

The phenomenon is similar to the one stated by Reynolds for demand uncertainty. If costs are high the marginal revenue from an additional capacity unit has opposite sign for a large and a small firm.

Corollary 1 : The equilibrium has the following properties.

(1) The equilibrium is necessarily asymmetric.

(2) For one firm the equilibrium quantity is larger than the Cournot quantity of a $c = 0$ firm. For the other firm the quantity is lower.

(3) The equilibrium pricing occurs in mixed strategies with probability $F\left(P(k_j)\right)$. With probability $1 - F\left(P(k_j)\right)$ the outcome is pure Bertrand competition $p = c$.

(4) For the mixed strategy equilibrium all properties of Kreps and Scheinkman hold.

5.1 Example

We derive the equilibrium of a natural example. Assume that costs are uniformly distributed between 0 and 1. We assume demand to be linear $D(p) = 1 - p$.

The Cournot reaction functions are $q_i(q_j) = \frac{1-q_j-c}{2}$ and so the weakly larger firm should build up capacities $k_i = \frac{1-k_j}{2}$. In equilibrium the smaller firm firm builds up capacities $k_j = \frac{1}{4}$ and the larger firm installs capacities $k_i = \frac{3}{8}$. In the second stage, both firms set prices equal to marginal costs if $c \geq \frac{3}{4}$. If costs are small, i.e. $c < \frac{3}{4}$ then both firms play fully mixed strategies. By Kovenock-Deneckere firm 2 earns the minmax profit. This minmax profit is equal to $\max\left(\frac{3}{32} + \frac{1}{6}c^2, 0\right)$. Please not that the support collapses to one point if $c = \frac{3}{4}$. In this case the firms would play $p = c = \frac{3}{4}$.

The equilibrium distribution could easily be computed like in lemma 6 of Kreps-Scheinkman. The cumulative density function of the equilibrium distribution obviously depends on the cost parameter c. As the uncertainty disappears before firms set prices the price setting behavior obviously depends on this new information.

The expected profit of the large firm is equal to $\frac{36}{1024}$, the expected profit of the small firm is equal to $\frac{24}{1024}$.

If capacity costs are positive, e.g. $b(k) = k^2$ equilibrium capacity is smaller. If we compare this to the monopoly result in section 2 we find a comprehensible result. We can see that aggregate quantity is higher in the duopoly case. Furthermore aggregate profits are also higher for the duopoly. The reason for that is simple. Due to convex capacity costs small firms are more efficient. But if we had increasing returns to scale at the capacity stage we could have the impressive result that a monopoly may be more efficient than an oligopoly, although we have constant marginal costs.

This leads to a very important instruction for competition policy. Competition authorities always have to take into account the ex ante capacity stage.

Though capacity costs are sunk at the price setting stage they have serious implications for the price setting subgame. The reason why one firm is so large may be the existence of an asymmetric equilibrium for the capacity stage.

From our point of view there is too much effort in analyzing actual price setting behavior. Competition authorities should always have a look at the larger game.

Competition authorities may conclude that a merger should be prohibited although a capacity pre-commitment stage induced a natural monopoly.

6 Extension

We know that in the full information case the result crucially depends on the assumption that costs are equal. This leads to the question whether this property is preserved under incomplete information.

Under incomplete information the problem is much more complicated. In order to keep things simple we assume that demand is linear $D(p) = 1 - p$. If the probability that both firms have a masspoint at the same price is zero, the profit function is

$$\begin{aligned} E\pi_i\left(p_i\left(c_i\right), c_i\right) &= \left(p_i - c_i\right)\min\left(D_i\left(p_i\right), k_i\right)\Pr\left(p_i < p_j\right) \\ &\quad + \left(p_i - c_i\right)\min\left(\max\left(D\left(p_i\right) - k_j, 0\right), k_i\right)\Pr\left(p_i \geq p_j\right) \end{aligned}$$

We know from lemma 1 that in the second stage at least one firm earns the minmax profit. The minmax profit is defined as

$$H_i(p, c_i, k_i) = \Pi(p^H)$$

where

$$p^H = \operatorname{argmax}_p \left(p_i - c_i\right)\min\left(\max\left(D\left(p_i\right) - k_j, 0\right), k_i\right)$$

We first have to analyze which firm earns the minmax profit. If both firms build up the same capacities the high cost firm will earn the minmax profit. The lower cost firm will earn the underbidder profit at $\underline{p}$. $\underline{p}$ is implicitly defined by $(p-c)k = H^*$. Hence, $\underline{p} = \frac{H^*}{k} + c$. But for the derivation of the equilibrium we cannot assume that capacities are necessarily equal. As in the last section we assume w.l.o.g. that firm i is weakly larger.

In the equal cost case we had only three cost regions. We will see now that if unit costs differ, there are much more different regions.

The pure strategy regions remain the same if capacities are small. If for both firms $k_i < r_{c_i}(k_j)$ then both firms charge the capacity clearing price $P(k_i + k_j)$. For our linear case we have that $c_i < 1 - 2k_i - k_j$ and $c_j < 1 - 2k_j - k_i$. In graph 6 we indicate the different regions for one firm. The region of capacity clearing pricing is A.

If costs of the undersold firm are too high it will not make any profit. This is the case if $D(c_i) > k_i$. This implies that if $c_i > 1 - k_i$, firm i will make zero profits whenever it is undersold. This is region C. Please note that if firm i has the lower cost it will make positive profits - even if costs are high.

In regions D and E firm i has the lower costs. This implies that firm i should earn the profit $L(\underline{p})$. But the costs of firm j are so high, that its minmax profit is 0. Hence, firm i will serve the whole market. Firm i now has to compute whether it is better to set the monopoly price or to set the highest price such that the competitor cannot undersell. The maximization problem is $\max(p - c_j)D(p)$ s.t. $p \leq c_j$. In region D the solution is $p_i = c_j$, in region E the solution is the monopoly price p^m.

The remaining two regions are the fully mixed strategy regions. In region B firm i has the higher costs and will therefore earn the minmax profit. In region F firm i will have a profit $(\underline{p} - c)$ min $(1 - \underline{p}, k_i)$. We have to stress that graph 6 is plotted for equal capacities. Surely, these lines will vary with varying capacities.

We are now able to give the theorem:

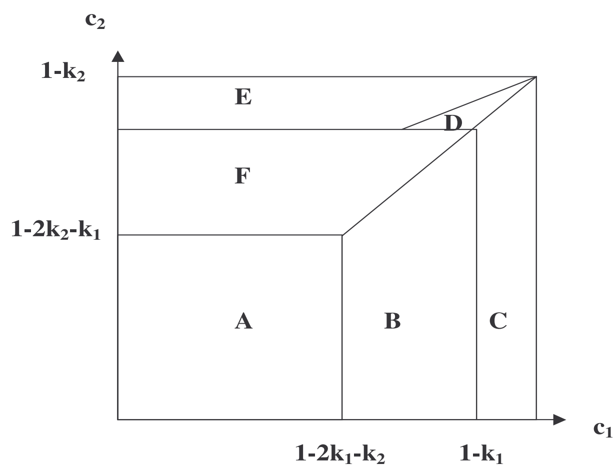

Figure 5: Outcomes in the Price Setting Game

Theorem 2 : The capacity precommitment game under uncertainty with different costs has a unique subgame perfect Nash Equilibrium:

Proof of Theorem 2 : From Kovenock Deneckere Theorem 1 we know that for each region the profit function is well behaved. The expected profit function is therefore concave and has a unique maximizer.

If we compute the reaction functions we find that reaction functions are not Cournot. We also find that aggregate quantities are lower. The reason for that is the same as in the full information case. If unit costs differ one firm has an advantage. There is a region where the low cost firm has a incentive to price the other firm out of the market. This yields to a competition effects that reduces capacities.

7 Conclusion

Typically, firms have to choose capacity a long time before they go the market. Between the capacity decision at the very beginning and the price decision on the market different influences may change parameters. Shocks on demand, capacity costs or marginal costs may alter the original maximization problem. Demand may be affected by consumer's preferences or suppliers of substitutes. Capacity costs may be changed by interest rates or e.g. a bad building ground for the factory. Marginal costs finally may be affected by the learn curve effect, other technology shocks, or organizational progress.

Through the discussion of the cost uncertainty case we learned that equilibria are asymmetric. Though the weakly larger firm chooses its capacities like a Cournot firm, the weakly smaller firm always has an incentive to reduce the capacity.

The most impressing result is that firms choose very high capacities. If capacity costs go to zero firms make positive profits if and only if they are in the fully mixed strategy region. A competitive outcome is possible if costs are high enough. Different from the Bertrand under uncertainty result (Spulber 1994) firms set prices equal to marginal costs with positive probability. But a considerable improvement is that with positive probability both firms make positive profits.

The existence of fully mixed strategies is important for the existing literature. In some markets we observe the well known Edgeworth cycles. Prices vary over a certain range without under- or overshooting the borders. We could argue that these cycles are consequences of fully mixed strategies or at least, that fully mixed strategies are the right proxy to explain this phenomenon. In Kreps-Scheinkman firms play pure strategies with probability 1. So the full information case cannot explain why such cycles should occur in a capacity pre-commitment setup. Therefore we can conclude from this paper that it is the phenomenon of uncertainty that may induce these Edgeworth cycles in the market for electricity.

References

[1] Basar, T., 1978, "Equilibrium Solutions in Static Decision Problems With Random Coefficients in the Quadratic Cost", IEEE Transactions on Automatic Control, AC-23(5), 960-962.

[2] Beckmann, M., 1967, "Edgeworth-Bertrand Duopoly Revisited" Rudolf Henn (ed.), Operations Research-Verfahren III, Anton Hain, Meisenheim am Glan, 55-68.

[3] Brown Kruse J., S. Rassenti, S. Reynolds, V. Smith 1994, "Bertrand-Edgeworth Competition in Experimental Markets , Econometrica, 62(2), 343-371.

[4] Boccard, N., X. Wauthy, 2000, "Bertrand Competition and Cournot Outcomes: Further Results" , Economics Letters, 68, 279-285.

[5] Budde, J.,R.F. Göx, 1999, "The Impact of Capacity Costs on Bidding Strategies in Procurement Auctions" , Review of Accounting Studies, 4, 5-13.

[6] Burdett,K., K.,Judd 1983, "Equilibrium Price Dispersion" , Econometrica, 51, 955-970.

[7] Chamberlin, E. H. 1933, "The Theory of Monopolistic Competition" , Harvard Economic Studies, XXXVIII, Harvard University Press, Cambridge.

[8] Dasgupta, P., Maskin E., 1986, "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory" , The Review of Economic Studies, 53, 1-26.

[9] Dasgupta, P., Maskin E., 1986, "Dasgupta P. E. Maskin :The Existence of Equilibrium in Discontinuous Economic Games, II: Applications" , The Review of Economic Studies, 53, 27-41.

[10] Davidson, C., Deneckere R., 1986, "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model" , The Rand Journal of Economics , 17, 404-415.

[11] Deneckere, R., Koveneock, D., 1996, "Bertrand-Edgeworth duopoly with unit cost asymmetry" , Economic Theory , 8, 1-25.

[12] Deneckere, R., Koveneock, D., 1992, "Price Leadership" , The Review of Economic Studies , 59, 143-162.

[13] Edgeworth, F.Y. 1925,"The Theory of Pure Monopoly" , in Papers Relating to Political Economy, 1, MacMillan,London.

[14] Fudenberg, D., J. Tirole, 1991, "Game Theory" , MIT Press, London.

[15] Gal-or E. 1986,"Information Transmission–Cournot and Bertrand Equilibria" , The Review of Economic Studies, 53(1), 85-92.

[16] Gibbons, R. 1992, "A Primer In Game Theory" , FT Prentice Hall.

[17] Klemperer, P. D. 1999, "Auction theory: A guide to the literature" , Journal of Economic Surveys, 13, 227-286.

[18] Kreps, D.,J. Scheinkman, 1983, "Quantity precommitment and Bertrand competition yield Cournot outcomes" , The Bell Journal of Economics , 14, 326-337.

[19] Krishna, V., 2002, "Auction Theory" , Academic Press, London.

[20] Levitan, B. Shubik, 1979, "Moral Hazard and Observability", Bell-Journal-of-Economics, 10(1), 74-91.

[21] Long van, N., A. Soubeyran, 2000, "Existence and Uniqueness of Cournot Equilibrium: A Constraction Mapping Approach", Economics Letters, 345-348.

[22] Mas-Collel, A. Whinston, J. Green 1995, "Microeconomic Theory" , Oxford University Press, New York.

[23] Maskin E., J. Tirole, 1988, "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs", Econometrica, 56(3), 549-569.

[24] Maskin, E., J. Riley, 1984, "Optimal Auctions with Risk Averse Buyers", Econometrica, 52, 1473-1518.

[25] Maskin,E. 2000, "Auctions, development, and privatization: Efficient auctions with liquidity-constrained buyers", European Economic Review 44, 667-681.

[26] Milgrom,P.,R.J. Webber: 1982, "The Value of Information in a Sealed Bid Auction", Journal of Mathematical Economics, 10, 105-114.

[27] Milgrom,P., R.J. Weber "Distributional Strategies for Games with Incomplete Information", Mathematics of Operations Research, 10, 619-632.

[28] Noel, M., "Edgeworth Price Cycles, Cost-based Pricing and Sticky Pricing in Retail Gasoline Markets", University of California at San Diego, Economics Working Paper Series 2004-04, Department of Economics, UC San Diego.

[29] Osborne,M.J.,Carolyn Pitchik, 1986, "Price Competition in a Capacity-Constrained Duopoly", Journal of Economic Theory, 38, 238-261.

[30] Reynolds,S.S., Bart J. Wilson, 2000, "Bertrand-Edgeworth Competition, Demand Uncertainty, and Asymmetric Outcomes", Journal of Economic Theory, 92, 122-141.

[31] Shapiro, C. 1986, "Exchange of cost information in oligopoly" , Review of Economic Studies, 53, 433-446.

[32] Spulber, D.F. 1995, "Bertrand Competition when Rivals' cost are Unknown" , The Journal of Industrial Economics, 43(1), 1-11.

[33] Vives, X., 1984, "Duopoly Information Equilibrium: Cournot and Bertrand" , Journal of Economic Theory, 34, 71-94.

[34] Vives, X., 2000, "Oligopoly Pricing: Old Ideas and new Tools" , MIT Press, London.