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On the Limitation of Warranty Duration*

by

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Abstract

This paper analyses the frequently observed phenomenon that firms offer product warranties which are of much shorter duration than the life expectancy of theses products. It is shown that competitive equilibria may entail limitation of warranty duration if firms face adverse selection problems with respect to different consumers.

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I Introduction

It is a well known empirical observation that the duration of warranties is much shorter than the life expectancy of the product they accompany. Given that warranties serve to insure consumers, limitation of warranty duration is a rather strange phenomenon: Why do firms insure consumers against the event of product break-down for a short time period while for the remainder of product life consumers bear all the risk alone? In a first-best world, risk neutral sellers will provide risk averse consumers with full insurance over the whole service life of a product[1)]. In order to explain warranties of limited duration one has to invoke imperfections which prevent the market from obtaining the first-best optimum.

Consumer product warranties commonly deny coverage if the product is put to commercial rather than domestic use[2)]. Accordingly, firms will seek to screen consumers who use the product at different intensities. This paper analyses how a market will allocate warranty contracts if firms cannot distinguish among groups of consumers using the product at different intensities. Competitive firms produce a homogeneous product with a stock of services which consumers continuously exhaust over time. The product may randomly break down during its service life. Quality is taken to be the probability that the product makes available total capacity. Firms face two groups of risk averse consumers: high-intensity users and low-intensity users. When utilized by high-intensity users, the product breaks down with certainty after one period. When utilized by low-intensity users, the product can last as long as two periods. Accordingly, high-intensity users seek insurance for one period whereas low-intensity users wish to be insured against the event of product break-down for two periods. Firms cannot distinguish between the two groups of consumers nor can they determine whether the product failed in period two because it was worn out by a high-intensity user or because an unlucky low-intensity user experienced a random failure. In this situation, high-intensity users will conceal their identity and buy low-priced insurance designed for low-intensity users if this turns out to be profitable.

Only two kinds of equilibria might arise under these circumstances. In a pooling situation both groups of consumers buy the same contract. In a separating situation the contract for low-intensity users is designed in such a way that high-intensity users prefer to purchase an offer designed for them. In section III and IV we analyse the sets of possible pooling and separating contracts, respectively. We show that although low-intensity users are able to obtain positive warranty coverage for both periods of product life, they will prefer contracts of limited duration, provided product quality is sufficiently high. Limited warranty duration means that there is no warranty coverage for period two, i.e. low-intensity users bear the risk of product failure alone. In a pooling situation low-intensity users reject any positive amount of second period insurance because this turns out to be too expensive. In a separating situation limitations on warranty duration serve the purpose of deterring high-intensity users from choosing the contract designed for low-intensity users. In section V we derive conditions under which contracts of limited duration constitute market equilibria.

The purpose of this paper is not to show merely that adverse selection leads to less than full insurance for low risk consumers[3)], but to characterize the time structure of the low risk consumers' warranty contracts in a two period insurance problem. We show that low-intensity users may not prefer warranty contracts with less than full, but positive insurance for both periods of product life. They may find it optimal to choose contracts which provide positive insurance for the first period and no insurance at all for the second.

Let us now briefly look in how far other existing theories are able to explain limitation of warranty duration. The exploitation theory, dating back to Kessler (1943) explains the design of warranties by the market power of manufacturers. According to this theory the terms of a warranty contract are drafted unilaterally by sellers to exploit consumers. This theory has the following difficulty in explaining limitation of warranty duration. Consider a risk neutral monopolist selling a commodity to consumers who wish to be insured in each period of product life. To exploit the surplus arising from the different attitudes towards risk, the monopolist will offer positive warranty coverage for each period of product life[4].

The signaling theory (Spence (1974), Grossman (1981)) rests on the following idea. Several sellers offer a product at different, exogenously given quality levels. Buyers cannot discern the respective quality of an individual offer. Because a more reliable product incurs lower warranty costs, a producer can signal a high quality of his product by an extensive warranty coverage. Obviously, the signaling theory predicts that firms producing more durable products signal this attribute by a warranty duration matching the life expectancy of the product[5].

The investment theory (McKean (1970), Priest (1981)) is based upon a moral hazard problem. In the way they handle a product, consumers exert some influence on the probability of product failure. The action chosen by consumers cannot be monitored by producers. The more warranty buyers get, the less incentive they have to avoid the event of product break-down. Consequently, in one period moral hazard situations warranty coverage has to be rationed to induce consumers to handle the product carefully. Yet, in particular repeated moral hazard situations there may be a countervailing effect. Consider a product which may hold for several periods. In each period the product either works satisfactorily or breaks down completely. Consumers enter the next stage of product life only when the product did not break down in the previous period. By their choice of effort consumers influence the probability of product failure. In each period they adjust their effort level optimally. Now, consider an increase in the warranty provided in case of product failure in the last period. Because such an increase raises the consumers' expected utility in the last period, it becomes more attractive to reach this stage of product life. Accordingly, it is worthwhile to be more careful in earlier periods. In this particular repeated situation we should therefore expect an increase of warranty coverage relative to a one period moral hazard problem due to the positive intertemporal incentive effect.

II The Model

Consider a set of identical, perfectly competitive firms selling a homogeneous product. The product has a maximum capacity which is normalized to one. The product may randomly break down after yielding total services v, $v \in [0,1]$. For $v < 1$ the break-down probability has density $f(v) > 0$ and $q = 1 - \int_0^1 f(v)dv \in (0,1)$ denotes the probability that the product makes available its total capacity; in the sequel we will refer to q as the quality of the product[6]. After total capacity is used up the product breaks down for sure. It is not worthwhile to repair a broken product.

Consumers wish to buy a single unit of the good in question. They exhaust the product's capacity over time which is measured by $t \in [0,\infty)$. The time of purchase is normalized to $T = 0$. There are two kinds of consumers: high-intensity users and low-intensity users with fraction $\lambda \in (0,1)$ of the population and $(1-\lambda)$, respectively. High-intensity users exhaust the product's capacity according to some exogenously given function $v = h(t) \geq 0$, where $\int_0^T h(t)dt$ denotes the number of services which are consumed in the time period $[0,T]$. The corresponding function for low-intensity users is given as $v = l(t) \geq 0$. High-intensity users have exhausted total capacity at some point in time T_h, where T_h is the minimum value of T such that $\int_0^{T_h} h(t)dt = 1$. Call the time span $[0,T_h)$ period one. At the end of period one, low-intensity users have only used up a fraction $\alpha = \int_0^{T_h} l(t)dt \in (0,1)$ of total capacity. They have exhausted the total stock of services at some point in time $T_l > T_h$, where T_l is the minimum value of T such that $1 - \alpha = \int_{T_h}^{T_l} l(t)dt$. Call the time span $[T_h, T_l)$ period two.

Consumers are risk averse. A product yielding total services v, $v \in [0,1]$ generates a monetary value of v for consumers. Consumers have no time preference, i.e. they only care about the number of services a product makes available and not about the time of consumption. Competitive pressure will force firms to sell the product at marginal production costs. We will assume that the consumers' willingness to pay for the product without any warranty exceeds marginal production costs for both groups. Therefore, all consumers wish to buy the product. Let M denote the consumers' wealth net of production costs. If the product is sold without any warranty, the expected utility for both groups of consumers is given by

$$EU = qU(M+1) + \int_0^1 U(M+v)f(v)dv,$$

where the utility function $U(\cdot)$ from $I\!R_+$ into $I\!R$ with $U'(\cdot) > 0$ and $U''(\cdot) < 0$ represents the consumers' risk preferences.

First, let us examine the case where firms can perfectly monitor the number of services $v = v_0$ which the product makes available before failing. At the time of purchase, risk neutral firms offer a monetary warranty schedule $w(v)$. Competition will force firms to offer the warranty at the fair odds rate $p\big(w(v)\big) = \int_0^1 w(v)f(v)dv$. Given this fair premium-benefit ratio, consumers will select full insurance, i.e.

$$\hat{w}(v) = 1 - v =$$

$$\underset{w(v)\geq 0}{\arg\max} \quad qU(M-p+1) + \int_0^1 U\big(M-p+v+w(v)\big)f(v)dv$$

$$\text{s.t.:} \quad p\big(w(v)\big) = \int_0^1 w(v)f(v)dv.$$

Note that the optimal warranty schedule $\hat{w}(v)$ decreases with v to equate marginal utility of income. The knowledge that consumers use the product at different intensities is redundant for firms. The indemnity is only conditional on v, independent of when the product breaks down.

Let us now assume that firms cannot observe v_0, the value taken by v when the product failed. They can only monitor whether the product has a defect at the end of period one or at the end of period two. They are furthermore not able to distinguish whether a broken product had a random failure during its service life or whether its capacity is exhausted. Under these circumstances, the warranty schedule $\hat{w}(v)$ is no longer incentive compatible. Each consumer who presents a broken product at the end of either period will pretend that his product failed without making available any services at all and thereby gain $\hat{w}(0) = 1$.

In this situation firms offer warranty contracts $\gamma = (w_1, w_2, p(w_1, w_2))$ which depend on the period in which the product failed, where $w_1 \geq 0$, $w_2 \geq 0$ denote the monetary warranty coverage in case the product breaks down in the first respectively second period; $p(w_1, w_2)$ denotes the price for the warranty vector. Consumers can claim either w_1 or w_2. To induce both groups of consumers not to claim w_2 with a product which broke down in the first period, firms can only offer warranty vectors (w_1, w_2) with $w_1 \geq w_2$.

Let us briefly consider the situation where firms are able to distinguish between the two groups. All low-intensity users who experience a random failure in period one will claim w_1. The probability of being unlucky within the first period is $\int_0^\alpha f(v)dv$. By the appropriate choice of the parameter $\beta \in (0,1)$ we will write $1 - q^\beta = \int_0^\alpha f(v)dv$ [7]. Those low-intensity users who are lucky in the first period and thereby enjoy α, but experience with probability $\int_\alpha^1 f(v)dv = q^\beta - q$ a random failure in the second period will claim w_2. When those low-intensity users who with probability q are lucky in both periods have used up total capacity, the warranty duration is over. High-intensity users who experience a random failure in period one with probability $(1-q)$ will claim w_1. Since firms know that high-intensity users have worn out the product in period one, they will not offer a positive second period warranty for this group. Accordingly, in a competitive equilibrium firms will offer two distinct sets of contracts at the respective fair odds rate for each group, i.e.

$$\Gamma_h = \{\gamma_h | w_1 \geq 0, w_2 = 0, p_h = (1-q)w_1\}$$

for high-intensity users and

$$\Gamma_l = \{\gamma_l | w_1 \geq w_2 \geq 0, p_l = (1-q^\beta)w_1 + (q^\beta - q)w_2\}$$

for low-intensity users. Both groups of consumers choose the best contract out of their respective set, i.e.

$$\hat{\gamma}_h = \arg\max_{\gamma_h \in \Gamma_h} EU(\gamma_h)_h = qU(M-p+1) + \int_0^1 U(M-p+v+w_1)f(v)dv \quad \text{and}$$

$$\hat{\gamma}_l = \arg\max_{\gamma_l \in \Gamma_l} EU(\gamma_l)_l =$$

$$qU(M-p+1) + \int_\alpha^1 U(M-p+v+w_2)f(v)dv + \int_0^\alpha U(M-p+v+w_1)f(v)dv.$$

Due to the strict concavity of $U(\cdot)$ and the fact that $f(v) > 0 \quad \forall v$, high-intensity users choose a positive warranty coverage for period one while low-intensity users choose a positive warranty coverage for period one and two. Accordingly, both groups of consumers prefer a warranty duration equal to the lifetime of the product.

Finally, consider the case where firms cannot distinguish between the two groups of consumers. High-intensity users may now opt to conceal their identity and also buy the contract $\hat{\gamma}_l$ designed for low-intensity users. Those high-intensity users who are lucky in the first period with probability q and enjoy total capacity will claim w_2 in the second period because their product is of no more use. Since firms cannot distinguish between a worn out product and a product which broke down randomly, they will also get w_2. Therefore, high-intensity users will either get w_1 with probability $(1-q)$ or w_2 with probability q.

Taking into account this incentive problem it is well known that only two kinds of outcomes might arise as competitive equilibria. In a pooling situation both groups of consumers buy the same contract; in a separating situation the contract for low-intensity users is designed in such a way that high-intensity users prefer to buy the offer designed for them. In the following we will look at both situations respectively. It will be shown that although under both circumstances low-intensity users are able to obtain positive warranty coverage for both periods, they may prefer contracts entailing limitation of warranty duration, i.e. $w_2 = 0$. We will then establish conditions under which contracts of limited warranty duration constitute market equilibria.

III Pooling Contracts

Pooling occurs if both groups buy the same contract $\gamma_p = \left(\bar{w}_1, \bar{w}_2, \bar{p}(\bar{w}_1, \bar{w}_2)\right)$. We will consider those pooling contracts which maximize the expected utility of low-intensity users subject to the aggregate break even constraint

$$\bar{p}(\bar{w}_1, \bar{w}_2) = [(1-\lambda)(1-q^\beta) + \lambda(1-q)]\bar{w}_1 + [(1-\lambda)(q^\beta - q) + \lambda q]\bar{w}_2. \qquad (1)$$

Among all offers on the market odds line 1 those contracts are of particular interest, because they are the only candidates which might persist in a competitive market; this will be shown in section V.

Consider the set of break-even pooling policies $\Gamma_p = \{\gamma_p | \bar{w}_1 \geq \bar{w}_2 \geq 0\}$ which allows for positive warranty coverage for both periods. We may now establish our first result that low-intensity users prefer a contract of limited duration, i.e. $\bar{w}_2 = 0$ out of this set, provided quality is sufficiently high. The proofs of Proposition 1 - 3 are relegated to the appendix.

Proposition 1: *There exists a quality level $q_c < 1$, such that for quality levels above this value, low-intensity users prefer within the set of pooling contracts Γ_p the one period pooling contract $\gamma_p^* = \left(\bar{w}_1^*, 0, \bar{p}(\bar{w}_1^*, 0)\right)$ which is a solution to the maximization problem*

$$\max_{\bar{w}_1} EU\left(\bar{w}_1, 0, \bar{p}(\bar{w}_1, 0)\right)_l$$

$$s.t.: \quad \bar{p}(\bar{w}_1, 0) = [(1-\lambda)(1-q^\beta) + \lambda(1-q)]\bar{w}_1 \quad .$$

This Proposition states that for high quality levels low-intensity users are only intersted in pooling contracts of limited duration. The intuition behind this result is as follows. The better the product, the higher is the high-intensity users' "second period break-down probability q" in relation to the low-intensity users' second period break-down probability $(q^{\beta} - q)$. Due to this large gap between second period break-down probabilities, low-intensity users will reject any positive amount of $\bar{w}_2$.

Proposition 1 establishes the existence of a critical quality level $q_c < 1$, such that for quality levels above this value low-intensity users prefer warranty contracts of limited duration. For this critical quality level we have the following comparative statics results.

Proposition 2:

i) q_c decreases with λ;

ii) q_c decreases with β;

iii) If $U(\cdot)$ exhibits increasing absolute risk aversion, q_c increases with α.

Proposition 2 i) can be understood as follows. The higher the fraction of high-intensity users, the more likely is the rejection of second period insurance by low-intensity users. Part ii) has the following interpretation. An increase in β means a shift of the low-intensity users' overall break-down probability $(1 - q)$ from period two into period one. Limitation of warranty duration is more likely to occur because period two pooling becomes more unfair. Part iii) can be interpreted as follows. The more capacity low-intensity users use up in the first period, the less likely is limitation of warranty duration.

Let us now consider under which conditions limitation of warranty duration occurs among the set of separating contracts.

IV Separating Contracts

A pair of contracts is said to be separating if each group of consumers prefers to buy the offer designed for them rather than the policy designed for the other group. By rationing the amount of warranty available at a low price and offering more coverage at a high price, firms induce high-intensity users to choose the high priced contract while low-intensity users purchase the low priced one. Thereby, consumers of different types reveal their identity. As will be shown in section V, competition will force firms to offer warranty coverage for high-intensity users at their fair odds rate $p_h = (1-q)w_1$. Given the fair premium-benefit ratio, high-intensity users will choose the contract $\hat{\gamma}_h$ which was derived in section II, generating an expected utility $EU(\hat{\gamma}_h)_h$.

Given a pair of distinct contracts, the offer designed for low-intensity users will be denoted by $\gamma_s = (\underline{w}_1, \underline{w}_2, p_l(\underline{w}_1, \underline{w}_2))$, with $p_l(\underline{w}_1, \underline{w}_2) = (1-q^\beta)\underline{w}_1 + (q^\beta - q)\underline{w}_2$. In the sequel we will refer to these offers for low-intensity users as separating contracts. To induce high-intensity users to choose their high priced offer $\hat{\gamma}_h$, separating contracts have to satisfy the following incentive constraint

$$EU(\gamma_s)_h = qU(M - p_l + 1 + \underline{w}_2) + \int_0^1 U(M - p_l + v + \underline{w}_1)f(v)dv \leq EU(\hat{\gamma}_h)_h.$$

Consider the set of break-even separating contracts $\Gamma_s = \{\gamma_s | \underline{w}_1 \geq \underline{w}_2 \geq 0, EU(\gamma_s)_h \leq EU(\hat{\gamma}_h)_h\}$ which allows for positive warranty coverage for both periods. We may now establish the result that low-intensity users prefer a warranty contract of limited duration, i.e. $\underline{w}_2 = 0$ out of this set, provided quality is sufficiently high.

Proposition 3: *For q sufficiently close to one, low-intensity users prefer within the set of separating contracts Γ_s the one period separating contract $\gamma_s^* = (\underline{w}_1^*, 0, p_l(\underline{w}_1^*, 0))$ defined by $EU(\gamma_s^*)_h = EU(\hat{\gamma}_h)_h$.*

This Proposition states that for high quality levels, low-intensity users are only interested in separating contracts of limited duration. The intuition behind this result is as follows. Consider a high-intensity user intending to buy a contract designed for low-intensity users. On the one hand, as q approaches one the difference between both groups' first period break-down probabilities $(1-q)$ and $(1-q^{\beta})$ becomes arbitrarily small. Since this advantage diminishes and his first period break-down probability becomes small, a high-intensity user would not care too much about giving up first period insurance.

On the other hand, as q approaches one, almost all high-intensity users would claim w_2, whereas almost no products of low-intensity users break down in the second period. The period two premium-benefit ratio would be very favourable for high-intensity users. Accordingly, only strong rationing of period two insurance could induce high-intensity users to choose the contract $\hat{\gamma}_h$.

So far we have shown that among the sets of pooling and separating offers which allow both for two period insurance low-intensity users may prefer contracts of limited duration. Let us now establish conditions under which contracts with this property constitute market equilibria.

V Equilibrium Results

In this section it is shown that the one period separating contract γ_s^* and the one period pooling contract γ_p^* can be supported as an equilibrium in the sense of Wilson (1977), provided quality is so high that both one period offers are the best choice for low-intensity users among the respective sets.

Definition: *A Wilson anticipatory equilibrium is a set of contracts such that, when consumers maximize their expected utility*

i) each contract in the equilibrium earns nonnegative profits and

ii) there is no set of contracts outside the equilibrium which earns positive profits in the aggregate and nonnegative profits individually, after the unprofitable policies in the original set have been withdrawn.

Consider first the case where low-intensity users prefer the one period separating to the one period pooling contract[8].

Proposition 4: *For q sufficiently close to one, if $EU(\gamma_s^*)_l \geq EU(\gamma_p^*)_l$ the set of contracts $\{\hat{\gamma}_h, \gamma_s^*\}$ constitutes a Wilson equilibrium*[9].

Proof: The above pair of contracts separates the two groups and each offer breaks even. It remains to be shown that there is no contract outside the equilibrium that can make positive profits.

According to Proposition 3, the one period separating contract γ_s^* is the best offer for low-intensity users among the set of separating contracts Γ_s. Hence, no firm is able to attract low-intensity users with another contract which does not violate the high-intensity users' incentive constraint.

The equilibrium candidate provides high-intensity users with their most preferred contract $\hat{\gamma}_h$ at their fair odds rate. No firm can, therefore, attract this group of consumers with an offer exclusively designed for them which does not make losses.

We still have to check whether a pooling contract can make positive profits. According to Proposition 1, the one period pooling offer γ_p^* is the best choice for low-intensity users among the set of break-even pooling contracts Γ_p. By the presumption of Proposition 4, low-intensity users prefer the one period separating contract γ_s^* to their best pooling offer γ_p^*. Thus, any firm offering a pooling contract would only attract high-intensity users and thereby incur losses.

Q.E.D.

Now consider the opposite case where low-intensity users prefer the one period pooling to the one period separating contract.

Proposition 5: *For q sufficiently close to one, if $EU(\gamma_s^*)_l < EU(\gamma_p^*)_l$, the contract γ_p^* constitutes a Wilson equilibrium*[10].

Proof: The equilibrium candidate γ_p^* is the best break-even pooling contract for low-intensity users; therefore, only contracts separating the two groups have a chance of making positive profits. Separating contracts out of Γ_s will not attract low-intensity users since they prefer the equilibrium candidate γ_p^* to their best offer γ_s^* out of this set.

Hence, only contracts using the existence of the pooling offer can attract low-intensity users. By offering warranty coverage which is less rationed than contracts out of Γ_s at a price slightly above their fair odds rate p_l, a firm can attract low-intensity users and make positive profits, while high-intensity users continue to buy the pooling offer. But left alone with high-intensity users the pooling contract makes losses and will be withdrawn. The new offer, being less rationed than contracts out of Γ_s, after the disappearance of the pooling contract will be bought by high-intensity users since it generates an expected utility larger than $EU(\hat{\gamma}_h)_h$. This will render the new contract unprofitable and under Wilson expectations it will not be offered.

Q.E.D.

VI Conclusions

Product warranties commonly deny coverage if the product is put to commercial rather than domestic use. Accordingly, firms providing warranties seek to screen consumers who use the product at different intensities. This paper analyses how a competitive market will allocate warranty contracts if firms cannot distinguish among groups of consumers using the product at different intensities. It is shown that this adverse selection problem is a possible explanation for limitation of warranty duration. Provided quality is sufficiently high, the existence of a group using the product at a higher intensity leads to limitation of warranty duration for the rest of the consumers.

We analyse the situation where firms can make warranty payments conditional on the period in which the product breaks down. In this situation low-intensity users are better off compared to the case where firms can only offer identical coverage for all periods. In this model such a combination of warranty coverages turns out to be a suboptimal choice for low-intensity users.

The purpose of the paper is to study the consequences of this adverse selection problem. Therefore, we have chosen a setup where consumers derive the same expected utility from the product in the absence of asymmetric information despite their different intensities of usage. If we add a positive consumers' time preference to the model, in the pooling situation limitation of warranty duration becomes more likely because low-intensity users care less about second period insurance. If the density of the break-down probability increases with time, in the pooling situation limitation of warranty duration becomes less likely since the low-intensity users' second period break-down probability is larger than in the setup we consider. Nevertheless, as long as the ratio of the low-intensity users' second to the first period break-down probability remains bounded as the overall failure probability becomes arbitrarily small, limitation of warranty duration as described will still occur in both, the pooling and the separating situation.

We consider quality as exogenously given. One might think of the quality level as a further screening device. Consider e.g. the situation where firms offer a high quality level with a one period warranty for high-intensity users and a lower quality level with (potential) two period insurance for low-intensity users . Due to the lower quality level of the second offer, the warranty levels necessary to induce high-intensity users to purchase the first offer are less rationed than in the quality pooling situation we consider[11)]. The analysis of this trade-off between a lower quality and a higher warranty remains an interesting topic for future research.

Appendix

Proof of Proposition 1:

When low-intensity users buy a pooling contract out of the set Γ_p their expected utility is given by

$$EU(\gamma_p)_l = qU(M-\bar{p}+1)+\int_{\alpha}^{1} U(M-\bar{p}+v+\bar{w}_2)f(v)dv+\int_{0}^{\alpha} U(M-\bar{p}+v+\bar{w}_1)f(v)dv. \quad (2)$$

By differentiating (2) with respect to p and $\bar{w}_2$, we obtain their marginal willingness to pay for a small increase in $\bar{w}_2$, i.e. their marginal rate of substitution between price and second period warranty:

$$\frac{dp}{d\bar{w}_2}(\bar{w}_1,\bar{w}_2)_l =$$

$$\frac{\int_{\alpha}^{1} U'(M-\bar{p}+v+\bar{w}_2)f(v)dv}{qU'(M-\bar{p}+1)+\int_{\alpha}^{1} U'(M-\bar{p}+v+\bar{w}_2)f(v)dv+\int_{0}^{\alpha} U'(M-\bar{p}+v+\bar{w}_1)f(v)dv}.$$

By differentiating again we further obtain $d^2p/d\bar{w}_2^2 < 0$.

If only one period pooling contracts $\big(\bar{w}_1, 0, \bar{p}(\bar{w}_1, 0)\big)$ are offered, low-intensity users will never demand $\bar{w}_1 > 1$, because the premium-benefit ratio is unfair and the certainty equivalent of the the second period lottery is less than one. In what follows we can therefore restrict our attention to the set of pooling contracts $\big\{\big(\bar{w}_1, 0, \bar{p}(\bar{w}_1, 0)\big) | \bar{w}_1 \in [0,1]\big\}$.

For a low-intensity user's marginal willingness to pay for an increase in $\bar{w}_2$ at the point $(\bar{w}_1, 0)$ we have the following estimate:

$$\frac{dp}{d\bar{w}_2}(\bar{w}_1,0)_l =$$

$$\frac{\int_{\alpha}^{1} U'(M-\bar{p}+v)f(v)dv}{qU'(M-\bar{p}+1)+\int_{\alpha}^{1} U'(M-\bar{p}+v)f(v)dv+\int_{0}^{\alpha} U'(M-\bar{p}+v+\underline{w}_1)f(v)dv} \leq$$

$$\frac{\int_{\alpha}^{1} U'(M-\bar{p}+v)f(v)dv}{qU'(M-\bar{p}+1)+\int_{\alpha}^{1} U'(M-\bar{p}+v)f(v)dv+\int_{0}^{\alpha} U'(M-\bar{p}+v+1)f(v)dv} \leq$$

$$\frac{\int_\alpha^1 U'(M-\bar{p}+\alpha)f(v)dv}{qU'(M-\bar{p}+1)+\int_\alpha^1 U'(M-\bar{p}+1)f(v)dv+\int_0^\alpha U'(M-\bar{p}+\alpha+1)f(v)dv} \leq$$

$$\frac{(q^\beta - q)U'(M-\bar{p}+\alpha)}{q^\beta U'(M-\bar{p}+1)+(1-q^\beta)U'(M-\bar{p}+\alpha+1)} \leq$$

$$\frac{(q^\beta - q)U'(M+\alpha)}{q^\beta U'(M+1)+(1-q^\beta)U'(M+\alpha+1)} \leq$$

$$\frac{(q-q^\beta)U'(M+\alpha)}{U'(M+1+\alpha)} \quad \forall \bar{w}_1 \in [0,1].$$

Low-intensity users will reject any movement towards a positive amount of $\bar{w}_2$ if marginal cost exceed their marginal willingness to pay; this is a sufficient condition as $d^2p/d\bar{w}_2^2 < 0$.

$$\frac{dp}{d\bar{w}_2}(\bar{w}_1,0)_l \leq \frac{(q^\beta - q)U'(M+\alpha)}{U'(M+1+\alpha)} < (1-\lambda)(q^\beta - q) + \lambda q \quad \Leftrightarrow$$

$$\Phi(q,\beta,\lambda) \equiv \frac{q^\beta - q}{(1-\lambda)(q^\beta - q)+\lambda q} < \frac{U'(M+1+\alpha)}{U'(M+\alpha)} \in (0,1). \tag{3}$$

As $\partial\Phi(\cdot,\beta,\lambda)/\partial q < 0$ for $q \in (0,1)$ and $\Phi(1,\beta,\lambda) = 0 \quad \forall \quad \lambda \in (0,1), \beta \in (0,1)$ we can conclude that there exists a critical quality level $q_c < 1$, such that for quality levels $q_c \leq q < 1$, low-intensity users reject any positive amount of $\bar{w}_2$. Thus, for q large enough they will prefer the one period pooling contract γ_p^*which maximizes their expected utility.

Q.E.D.

Proof of Proposition 2:

i) Follows from the fact that $\partial\Phi(q,\beta,\cdot)/\partial\lambda < 0$ for $q > 1/2$ with $\Phi(q,\beta,\lambda)$ defined by (3);

ii) follows from the fact that $\partial\Phi(q,\cdot,\lambda)/\partial\beta < 0$;

iii) follows from the observation that the right hand side of (3) decreases with α if $U(\cdot)$ exhibits increasing absolute risk aversion.

Q.E.D.

Proof of Proposition 3:

Consider the equality in the incentive constraint

$$EU(\gamma_s)_h = EU(\hat{\gamma}_h)_h \tag{4}$$

This equality implicitly defines an indifference curve in the (w_1, w_2)-space, such that all warranty combinations $(\underline{w}_1, \underline{w}_2)$ on this curve with the corresponding price $p_l(\underline{w}_1, \underline{w}_2)$, are as good for high-intensity users as the contract $\hat{\gamma}_h$. By totally differentiating (4) we obtain the slope of this indifference curve:

$$\frac{d\underline{w}_1}{d\underline{w}_2}\Big|_h = -\frac{qU'(M - p_l + 1 + \underline{w}_2) - \partial p_l/\partial \underline{w}_2 EU'(\gamma_s)_h}{\int_0^1 U'(M - p_l + v + \underline{w}_1)f(v)dv - \partial p_l/\partial \underline{w}_1 EU'(\gamma_s)_h}.$$

First note that $\frac{d\underline{w}_1}{d\underline{w}_2}\big|_h < 0$ for q sufficiently close to one. For the denominator we have the following estimate.

$$\int_0^1 U'(M - p_l + v + \underline{w}_1)f(v)dv - (1 - q^\beta)EU'(\gamma_s)_h =$$

$$q^\beta \int_0^1 U'(M - p_l + v + \underline{w}_1)f(v)dv - (1 - q^\beta)qU'(M - p_l + 1 + \underline{w}_2) \geq$$

$$q^\beta(1 - q)U'(M - p_l + 1 + \underline{w}_1) - (1 - q^\beta)qU'(M - p_l + 1 + \underline{w}_2) \geq$$

$$(1 - q)U'(M - p_l + 1 + \underline{w}_1) - (1 - q^\beta)U'(M - p_l + 1 + \underline{w}_2).$$

This expression is positive as $\underline{w}_1 \geq \underline{w}_2$.

For the nominator we have the following estimate:

$$qU'(M - p_l + 1 + \underline{w}_2) - (q^\beta - q)EU'(\gamma_s)_h \geq$$

$$qU'(M - p_l + 1 + \underline{w}_2) - (q^\beta - q) \cdot [qU'(M - p_l + 1 + \underline{w}_2) + (1 - q)U'(M - p_l + \underline{w}_1)] >$$

$$qU'(M - p_l + 1 + \underline{w}_2) - (q^\beta - q)U'(M - p_l + \underline{w}_1).$$

This expression is positive provided that

$$\frac{U'(M-p_l+1+\underline{w}_2)}{U'(M-p_l+\underline{w}_1)} > \frac{q^\beta-1}{q} = q^{\beta-1}-1,$$

which is true for q large enough.

Given that $\frac{d\underline{w}_1}{d\underline{w}_2}|_h < 0$ for q sufficiently close to one we find by inspection that

$$\lim_{q\to 1}\frac{d\underline{w}_1}{d\underline{w}_2}|_h = "-\infty".$$

Now take the one period separating contract for low-intensity users γ_s^* defined by (4). Consider the set of contracts $(w_1, w_2, p_l(w_1,w_2))$, from which low-intensity users get the same expected utility as from γ_s^*. The set is defined by

$$EU(\gamma_s^*)_l = qU(M-p_l+1)+\int_\alpha^1 U(M-p_l+v+w_2)f(v)dv+\int_0^\alpha U(M-p_l+v+w_1)f(v)dv. \tag{5}$$

Equation (5) implicitly defines an indifference curve for low-intensity users in the (w_1, w_2)-plane through $(\underline{w}_1^*, 0)$. By totally differentiating (5) we obtain the slope of this indifference curve.

$$\frac{dw_1}{dw_2}|_l = -\frac{\int_\alpha^1 U'(M-p_l+v+w_2)f(v)dv-(q^\beta-q)EU'(w_1,w_2,p_l)_l}{\int_0^\alpha U'(M-p_l+v+w_1)f(v)dv-(1-q^\beta)EU'(w_1,w_2,p_l)_l}.$$

Here we have the following estimate:

$$\frac{dw_1}{dw_2}|_l > -\frac{q^\beta-q}{1-q^\beta}\cdot\frac{U'(M-p_l+1+w_2)-EU'(w_1,w_2,p_l)_l}{U'(M-p_l+w_1)-EU'(w_1,w_2,p_l)_l} \tag{6}$$

if the expression is negative and

$$\frac{dw_1}{dw_2}|_l < -\frac{q^\beta-q}{1-q^\beta}\cdot\frac{U'(M-p_l+\alpha+w_2)-EU'(w_1,w_2,p_l)_l}{U'(M-p_l+\alpha+w_1)-EU'(w_1,w_2,p_l)_l} \tag{7}$$

if the expression is positive. Let us now consider the limits of (6) and (7) resp., for q converging to one. The limit of the first factor of (6) and (7) can be calculated by L'Hôpital's rule:

$$\lim_{q\to 1} -\frac{q^\beta - q}{1-q^\beta} = \lim_{q\to 1} -\frac{\beta q^{\beta-1}-1}{-\beta q^{\beta-1}} = -\frac{1-\beta}{\beta},$$

which is finite. The limit of the respective second factor of (6) and (7) is also finite. Hence, we can conclude that

$$\lim_{q\to 1} \frac{dw_1}{dw_2}\Big|_l \quad \text{is finite.}$$

For q sufficiently large, we have shown that the two indifference curves defined by (4) and (5) only intersect once in $(\underline{w}_1^*, 0)$. For any $w_2 > 0$ the curve of incentive compatible contracts lies strictly underneath the low-intensity users' indifference curve. Thus, the one period separating contract γ_s^* is the best choice for low-intensity users among the set of incentive compatible contracts.

Q.E.D.

Footnotes

1.) See Brown (1974).

2.) See Priest (1981).

3.) See e.g. Rothschild and Stiglitz (1976) or Wilson (1977) who establish this result for a one period insurance problem.

4.) See also Heal (1977) who shows for a one period insurance problem that a risk neutral monopolist provides risk averse consumers with full insurance.

5.) The same line of argument applies to the closely related theory which views warranties as an incentive device for firms to produce high quality products. Rather than taking the characteristics of products as fixed these models endogenize the quality choice by firms. See e.g. Spence (1977).

6.) The value of q captures one aspect of the product's quality characteristics. One can think of other values such as the expected number of services a product makes available, i.e. $E(v) = q + \int_0^1 v f(v) dv$. Products with the same q but different densities of the breakdown probabilities in general exhibit different expected numbers of services. Yet it is the probability that the product makes available its total capacity which will play an important role in our analysis.

7.) This representation implies a special case of the general condition we need to prove our results, namely that $\forall \alpha \in (0,1)\ \exists K_\alpha$ such that $\forall q \in (0,1) \quad \left[\int_\alpha^1 f(v)dv / \int_0^\alpha f(v)dv\right] < K_\alpha$. This last condition has the economic interpretation that if q becomes large, low-intensity users do not become too 'similar' to high-intensity users. See in particular the proof of Proposition 3.

8.) We adopt the convention that given low-intensity users are indifferent between the separating and the pooling contract, they choose the separating one.

9.) Note that the set of contracts $\{\hat{\gamma}_h, \gamma_s^*\}$ in fact constitutes a Nash equilibrium in the sense of Rothschild and Stiglitz (1976).

10.) Note that if $EU(\gamma_s^*)_l < EU(\gamma_p^*)_l$ the pooling contract γ_p^* is the only stable equilibrium in the sense of Kohlberg and Mertens (1986) for the following three stage game. At stage one, firms offer contracts. At stage two, consumers choose among these contract offers. At stage three, firms may reject whatever contract applications they have received at stage two. See Hellwig (1987).

11.) See e.g. Matthews and Moore (1987) who analyse how a monopolist can use quality, warranty and price to screen buyers who have different preferences for a product.

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